A real-time picking and sorting system in e-commerce distribution centers

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Abstract

Order fulfillment is the most expensive and critical operation for companies engaged in e-commerce. E-commerce distribution centers must rapidly organize the picking and sorting processes during and after the transaction has taken place, with the ongoing need to create greater responsiveness to customers. Sorting brings a relatively large setup time, which cannot be well admitted by existing polling models. We build a new stochastic polling model to describe and analyze such systems, and provide approximate explicit expressions for the complete distribution of order line waiting time for polling-based order picking systems and test their accuracy. These expressions lend themselves for operations and design operations, including deciding between "pick-and-sort" or "sort-while-pick" processes, and warehouse performance evaluation.

Keywords: picking and sorting system, warehousing, e-commerce distribution centers, facility logistics

1 Introduction

Order fulfillment, "the last mile of e-commerce", is among the most crucial elements of e-commerce, and "the most expensive and critical operation" for companies engaged in e-commerce (Page 54, [1]). In e-commerce environment, customers who can order with the ease of a click of a button, expect delivery in the same fast and easy way ([2], [3]). The primary challenge that e-commerce distribution centers are facing is the radical reduction of order-to-delivery time to customers since customers tend to make a higher number of order transactions(for demand characteristics of online orders, see [4]), in smaller order size (see [5] and [4]). For example, Xue et al. [6] report that the proportion of orders with single order line is 64% -66% in off-peak season and 56% in peak season for a large online retailer, dominant in both seasons. Companies tend to accept late orders while providing rapid and timely delivery within tight time windows. The main disadvantage of online retailers, compared with traditional retailers, is immediacy ([7]): While the product can immediately be taken home after purchasing in a physical store, the customer must wait for the shipment to arrive in the case of e-commerce. Besides, most customers can not see the real products before purchasing, the probability that the customer will cancel the order is higher than the traditional purchasing process and so the fluctuation of customer demand is highly significant (see [4]). Moreover, online retailers face higher pressure from sensitive customers since internet allows customers change to another different web site with a single click, unlike the laborious process of selecting an alternate brick-and-mortar traditional retailer. This pressure is also because online customers consider speedy delivery as a significant symbol of trust ([6]). Therefore, Lee and Whang [1] think the ability to fulfill and deliver orders on time could determine an online retailer's success.

Facing these challenges, some e-commerce distribution centers adopt personalized material flows in order to achieve a fast-response competitive advantage. [8] report that, in Nettimarket (nettimarket.com), an online retailer in Finland, "45 minutes is calculated for personal service per order. This includes the whole process from obtaining the order to delivering the goods to the customer's doorstep". Even some e-commerce giants are trying to boost sales volumes by speedy order fulfillment. In October 2009, Amazon began to offer "same-day delivery" to selected cities including New York, Washington, Philadelphia, and LA. This new option, called "local express delivery", allows Amazon's subscribers receive shipments on the same day they place an order. AirNet, a large U.S. logistics company, offers "same-day delivery" 24 hours a day and 7 days a week in a larger region, including 100 major cities in U.S. Obviously, managing order picking systems effectively and efficiently is a challenging process in these distribution centers. A prime objective is to shorten throughput times for order picking, and to guarantee the meeting of due times for shipment departures. In this paper, we particularly concern the waiting time in warehouse operations since as pointed out by J.P.Morgan, the step Amazon is taking is actually to shorten customers' waiting time between order and delivery (see zdnet.com).

Lee and Whang [1] summarize five e-fulfillment (online order fulfilment) strategies: logistics postponement, dematerialization, resource exchange, leveraged shipment and clicks-and-mortar. This paper mainly considers a new strategy, the application of real-time system to online order fulfillment. With this strategy, the order and operation information are highly synchronized to achieve unprecedented fulfillment productivity and customer service. For an introduction of this strategy, see [9] and "e-fulfillment strategy" presented by [4]. Several companies are pioneers of this strategy. In February 2010, Intelligrated, a leading American automated material handling solution provider headquartered in Cincinnati, announced to re-launch a "real time order fulfillment system", a comprehensive suite including pick-to-light, pick-to-voice, pick-to-cart (a mobile order picking solution), RF picking and an order fulfillment software (see intelligrated.com or moreRFID.com). Colchester, a wine company in UK, is using a real-time order fulfillment system to expedite shipments throughout UK (see ITshowcase.co.uk).

Batching picking and sorting is a traditional way to organize the fulfilling process in case of a large number of daily order lines. However, batch formation takes time and, as the number of daily orders to be processed increases and as the required lead time becomes shorter, there may be more efficient ways to organize the order picking process. Gong and de Koster [10] propose a way, Dynamic Picking System (DPS), to organize it by using real-time order fulfillment strategy. In a DPS, orders arrive online and are picked in batch, followed by later sorting per customer order (a pick-and-sort system, see [11]). The picker travels the entire or a part of the warehouse and picks all outstanding order lines in one pick route to a pick cart. During a pick cycle, pick information is constantly updated by a pick-by-light, pick-by-RFID, pick by handheld terminal, or voice-picking system. Compared with static picking, where the pick locations during a pick cycle are given and fixed, dynamic picking can shorten the response time and can thereby improve the customer service. For example, "China online shopping", an online retailer in Southern China, is using such a system.

Sorting, according to [4], is a crucial warehouse operation for e-commerce distribution centers since smaller online order sizes induce heavier workload and increase difficulty to sort and accumulate items for individual shipment. We consider real-time picking and sorting systems (RPSS) in this paper. Currently there is no good way of analyzing the performance of a RPSS. Sorting brings a large setup time, which cannot be admitted by existing polling models. We build a new stochastic polling model to describe and analyze such systems, and find closed-form expressions for the order line waiting times. Since the response time of a real-time system is very short, it imposes a high requirement on fast algorithms. We delivery a high-speed algorithm to fit this special operational requirement. In practice, managers need to allocate sorting time, either to storage positions, which is called as "sort-while-pick", or in the depot, which is named as "pick-and-sort". As the knowledge of authors, we cannot find very well existing methods to determine sorting policies in real-time order picking environment.

The contribution of this paper are as follows: (1) We provide approximate explicit expressions for the complete distribution of order line waiting time for polling-based order picking systems. (2) These expressions lend themselves for operations and design operations, including deciding between "pick-and-sort" or "sort-while-pick" processes, and warehouse performance evaluation. (3) We provide a fast algorithm which can satisfy the requirement of real-time operations.

The remainder of the paper is organized as follows. In the following section, we conduct a brief literature review. We introduce the problem description and stochastic polling models for a RPSS in Section 3. Section 4 is devoted to the analysis of models, mainly including closed-form expressions for the performance measurement of a RPSS. In Section 5, we present our numerical experiments, verify the analysis, compare different policies, and show the advantage of polling system over traditional batch picking. We conclude with final comments in Section 6.

2 Literature review

Online order fulfillment (also referred as e-fulfillment) is an active research topic in both management information systems, and operations management fields and studied by qualitative, empirical, and modeling methods. (1) Online order fulfillment began with qualitative study. Andeson and Lee [12] conduct a thorough discussion of e-fulfillment issues from the first click to the last mile, i.e., order fulfillment. Lee and Whang [1] present main e-fulfillment strategies. Tarn et al. [4] study customer and demand characteristics of online orders, which support some assumptions in this paper. (2) Empirical study on online order fulfillment is relatively rich. Pyke et al. [13] examine the challenges of e-fulfillment in Internet furniture industry and provide lessons for online retailers. Accenture reported problems and situation of U.S. e-fulfillment in Christmas 2000 (see accenture.com), and found 12% orders were not received in time, 67 % deliveries were not received as ordered. These data are one of important reasons to draw early attention on the study of online order fulfillment. (3) We can find few modelling studies on online order fulfillment. Among these researches, Xu et al. [6] build a network flow model to examine the benefits of periodically re-evaluating the real-time order fulfillment decisions.

Picking accounts for the most proportion of order fulfillment cost (see [4] and [14]). Two major types of order picking systems can be distinguished: parts-to-picker and picker-to-parts systems (for a literature review, see [11]). It is also possible to distinguish picker-to-parts systems by the order arrival and release. This can be either deterministic and planned ([15]) or online and stochastic ([16]). Only few papers study online order arrivals ([17], [18], and [19]).

Sorting is another important operation in order fulfillment (see [4]). Manual picking and sorting system includes two operation protocols, “sort-while-pick” and “pick-and-sort”. Van Nieuwenhuyse and de Koster [18] have compared “sort-while-pick” and “pick-and-sort” in 2-block warehouses in a batch picking environment. Automated sortation is also widely applied. Johnson [20] has studied the impact of sorting strategies on automated sortation system performance.

The processes with online order arrivals and processing can be described by a so-called polling model: a system of multiple queues accessed in cyclic or other specified sequence by a single server. There is considerable literature on polling models, which are used to model an abundant set of systems like computer and telecommunications networks ([21]). There are also relevant applications in operations management. For instance, Koenigsberg and Mamer [22] consider an operator who serves a number of storage locations on a rotating carousel conveyor. Bozer and Srinivasan [23] consider tandem configurations for automated guided vehicle systems and analyze single-vehicle loops. Bozer and Park [24] present single-device, polling-based material handling systems.

3 Model

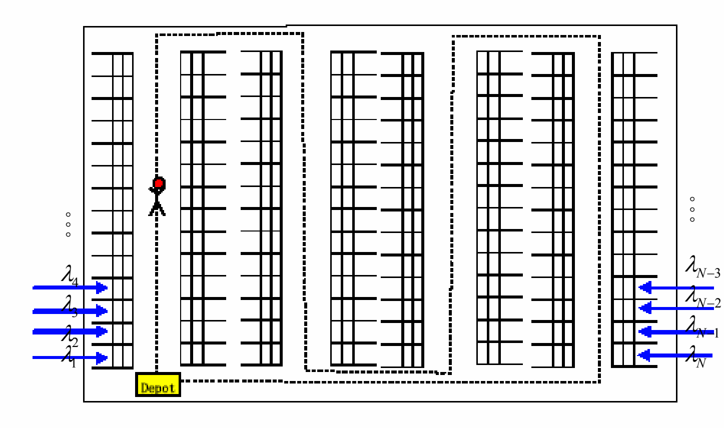

Figure 1: Top view of a RPSS

We consider RPSS in a general parallel-aisle warehouse with K aisles and L storage positions at each side of one aisle (see Figure 1). Without loss of generality, we assume the number of aisles is even. Order pickers pick the items following a two-sided picking policy, which means they pick at both sides in one pass, so called two-sided picking, and therefore we view the storage positions opposite to each other at both sides of an aisle as one position. We assume the picking cart is uncapacitated. In the case of e-commerce distribution centers, this is usually not a real restriction, as the route often finishes before the cart is full. The server here represents the order picker and the queues correspond to corresponding storage positions, each containing a different product.

The arrival order line stream at position i (each representing a certain product) is a Poisson process with rate $\lambda_i, 1 \leq i \leq N$. We also assume there is no replenishment required in one picking cycle, and each queue is assumed to have infinite buffer capacity, as usual in the polling literature. At each queue, order lines are served on a First-Come-First-Serve basis. The picking times are assumed to be IID random variables with finite first and second moments. Let $\beta_i^{(h)}$, where h^{th} denotes the h^{th} moment of the service times at position $i, i = 1, ...N, h = 1, 2$.

The storage positions are visited according to a strict S-shape routing policy in a cyclic sequence, which means that any aisle is traversed entirely. Since the position and order information is updated in real-time, the order picker cannot choose not to enter some aisles as might be possible in a static picking system. From the last visited aisle, the order picker returns to the pick position 1 via the depot: the depot is on the path from the position to the position 1.

This routing policy is associated with our layout. This paper mainly considers a parallel-aisle one block warehouse. In real-time picking and sorting systems, since a picker must traverse all storage positions, a middle aisle in a two-block warehouse not only cannot shorten the traveling distance by creating shortcuts, but also creases traveling distances for the width of middle aisle. With a strict S-shape traveling policy, the performance of real-time picking and sorting system in a one-block warehouse is better than that in a two-block warehouse.

The service times at queue i are independent, identically distributed random variables. When the server starts service at queue i, a deterministic setup time is incurred. The total setup time t in a cycle is a sum of the setup time at positions. The setup time could be traveling time and sorting time at the depot, dependent on protocols. In RPSS, a setup is always incurred even if the subsequent queue is empty.

The occupation rate ρ_i (excluding setups) at queue i is defined by $\rho_i = \lambda_i \beta_i$ and the total occupation rate ρ is given by $\rho = \sum_{i=1}^{N} \rho_i$. Now, $\rho < 1$ and $t < \infty$ constitute necessary and sufficient stability conditions for any nonidling policy (for further study on stability conditions, see e.g., [25]). In the remainder of the present paper, these stability conditions are assumed to hold as we restrict the attention to steady-state behavior.

The performance measure of interest is the delay W_i of a type-i customer, $i = 1, 2, \ldots, N$, in case the setup times tend to infinity. Since the delay grows to infinity in the limiting case, we focus on the asymptotic scaled delay $\frac{W_i}{t}$ as $t \to \infty$, where the ratios of the setup times remain constant, $i = 1, 2, \ldots, N$. Of course, our results for the delay distribution can be readily translated into results for the queue length distribution via the distributional form of Little's law.

(1)Pick-and-sort

After picking all the items, the order picker drops off the picked items at the depot, and also sort and transport them. The picker's sorting time is independent of the travel time. The picker will sort all orders at the depot, which lead to a sorting time τ_s. This is a polling system consisting of N queues and a large setup time, attended by a single order picker.

Let $t_i^{(h)}$ with $i = 1, ..., N-1$ denote the h^{th} moment of the travel times from queue i to queue $i+1$, $t_N^{(h)}$ denotes the h^{th} moment of the travel time from the position N to the position 1, and the sorting time is denoted with the first and second moments (τ_s, ξ_s). When the order picker travels from one aisle k to the next one $k+1$, the travel time will be larger than that between in-aisle locations. The first and second moments are denoted by (τ_1, ξ_1) .When the order picker travels from the last position back to the position 1, the travel time is largest with the first and second moments (τ_2, ξ_2) . Within one aisle, the times needed by the order picker to travel from one queue to the next are assumed to be IID random variables with finite first two moments (τ_3, ξ_3). We therefore have:

$$t_i^{(1)} = \tau_1, i = Lk, k = 1, ..., (K-1); \tau_2, i = N; \tau_3, otherwise \tag{1}$$

The mean of the total travel and sorting operation time t during a cycle of picking is respectively specified by,

$$t = \sum_{i=1}^{N} t_i^{(1)} + \tau_s = \tau_1(K-1) + \tau_2 + \tau_3(L-1)K + \tau_s. \tag{2}$$

let τ_t to denote the total **t**ravel time $\sum_{i=1}^{N} t_i^{(1)}$. The interarrival times of orders are assumed to be independent of the picking times, travel times and sorting time of the order picker. The traffic load to queue i is defined by $\rho_i = \lambda_i \beta_i^{(1)}, 1 \le i \le N$, and the total traffic load of the system is $\rho = \sum_{i=1}^{N} \rho_i$.

In a pick-and-sort protocol, the service time is the picking time, and setup time S is a sum of traveling time and a sorting time τ_s at the depot. Let picking time be μ_i at position i, the service time $\beta_i = \mu_i$.

(2)Sort-while-pick

In this protocol, the picker will sort orders at each position, which lead to a sorting time s_i at each position i. We take the same assumptions with "pick-and-sort" protocol in arrival processes, replenishment, infinite buffer capacity, picking times, and picking routing policy. The traffic load to queue i is defined by $\rho_i = \lambda_1 \beta_1^{(1)}, 1 \le i \le N$, and the total traffic load of the system is $\rho = \sum_{i=1}^{N} \rho_i$.

The difference is mainly in service time and total travel operations time t since the service time $\mathbb{E}[B_i]$ is the picking and sorting time, and the setup time t is a sum of traveling time in a sort-while-pick protocol. (1)Let $\beta_i^{(h)}$, where h^{th} denotes the hth moment of the service times at position $i, i = 1, ...N, h = 1, 2$. In a "sort-while-pick " protocol, $\beta_i^{(h)} = \mu_i^{(h)} + s_i^{(h)}$. (2) The total travel operation time t during a cycle of picking are respectively specified by,

$$t = \sum_{i=1}^{N} t_i^{(1)} = \tau_1(K-1) + \tau_2 + \tau_3(L-1)K \tag{3}$$

.

(3) Picking service policies

At each queue, the picking service policy prescribes how the order lines (if any) at each location should be picked. Throughout the present subsection we discuss some basic results of branching-type polling systems. A large number of service policies have been considered in the past polling research. We mainly consider exhaustive discipline, i.e., a queue must be empty before the server moves on, and gated discipline i.e., only those customers in the queue at the polling instant are served.

4 Analysis

In Section 4.1, we analyze a RPSS with "pick-and-sort" protocol. In Section 4.2, we analyze a RPSS with "sort-while-pick" protocol. We consider the deterministic sorting time in both cases.

4.1 RPSS with "pick-and-sort" protocol

Based on a given LST of the waiting time distribution of a type-1 order line $\mathbb{E}[e^{-\omega W_1^0}]$, Winands [26] obtains the LST of the distribution of the asymptotic scaled orderline waiting time $\mathbb{E}[e^{-\omega \frac{W_1}{t}}]_{(t \to \infty)}$, in combination with the convergence

Table 1: Performance evaluation of the RPSS with "pick-and-sort" policy

Picking policy	Distribution of the order line waiting time	Mean	The second moment
Exhaustive	$U[0, \frac{1-\rho_1}{1-\rho}]t$	$\frac{1-\rho_1}{2(1-\rho)}t$	$\frac{1}{3}\left(\frac{1-\rho_1}{1-\rho}\right)^2 t^2$
Gated	$U[\frac{\rho_1}{1-\rho}, \frac{1}{1-\rho}]t$	$\frac{1+\rho_1}{2(1-\rho)}t$	$\frac{1-\rho_1^3}{3(1-\rho)^2(1-\rho_1)}t^2$

Notes: $\beta_i = \mu_i$,$\rho = \sum_{i=1}^N \lambda_i\mu_i$, $\rho_1 = \lambda_1\mu_1$,$t = \sum_{i=1}^N t_i^{(1)} + \tau_s = \tau_1(K-1) + \tau_2 + \tau_3(L-1)K + \tau_s$.

Table 2: Performance evaluation of the RPSS with "sort-while-pick" policy

Picking policy	Distribution of the order line waiting time	Mean	The second moment
Exhaustive	$U[0, \frac{1-\rho_1}{1-\rho}]t$	$\frac{1-\rho_1}{2(1-\rho)}t$	$\frac{1}{3}\left(\frac{1-\rho_1}{1-\rho}\right)^2 t^2$
Gated	$U[\frac{\rho_1}{1-\rho}, \frac{1}{1-\rho}]t$	$\frac{1+\rho_1}{2(1-\rho)}t$	$\frac{1-\rho_1^3}{3(1-\rho)^2(1-\rho_1)}t^2$

Notes: $\beta_i = \mu_i + s_i$,$\rho = \sum_{i=1}^N \lambda_i(\mu_i + s_i)$,
$\rho_1 = \lambda_1(\mu_1 + s_1)$,$t = \sum_{i=1}^N t_i^{(1)} = \tau_1(K-1) + \tau_2 + \tau_3(L-1)K$.

theorem of Feller for Laplace-Stieltjes Transforms, we have, the distribution of the asymptotic scaled order line waiting time, which is uniformly distributed. In the analysis, we use an approximation method, which is carried out for W/t, followed by a rescaling to express order line waiting time W. We can also obtain the moments of the asymptotic scaled order line waiting time, and therefore calculate performance evaluation of the RPSS with pick-and-sort and deterministic sorting time in Table-1. For example, for exhaustive policy, its distribution of order line waiting time is a uniform distribution with $[0, \frac{1-\lambda_1\mu_1}{1-\sum_{i=1}^N \lambda_i\mu_i}(\sum_{i=1}^N t_i^{(1)} + \tau_s)]$, with a mean $\frac{1-\lambda_1\mu_1}{2(1-\sum_{i=1}^N \lambda_i\mu_i)}(\sum_{i=1}^N t_i^{(1)} + \tau_s)$, and the second moment $\frac{1}{3}\left(\frac{1-\lambda_1\mu_1}{1-\sum_{i=1}^N \lambda_i\mu_i}\right)^2 (\sum_{i=1}^N t_i^{(1)} + \tau_s)^2$.

4.2 RPSS with "sort-while-pick" protocol

We study the asymptotic order line waiting time in a RPSS system with deterministic distributed setups under heavy traffic. From [26], we have the distribution of the asymptotic scaled order line waiting time, which is also uniformly distributed. We thereby obtain the moments of the asymptotic scaled order line waiting time.

Based on this distribution, we calculate performance evaluation of the RPSS with sort-while-pick and deterministic sorting time in Table-2. For example, in case of the exhaustive picking policy, the order line waiting time is uniformly distributed on $[0, \frac{1-\lambda_1(\mu_1+s_1)}{1-\sum_{i=1}^N \lambda_i(\mu_i+s_i)}(\sum_{i=1}^N t_i^{(1)})]$, with a mean $\frac{1-\lambda_1(\mu_1+s_1)}{2(1-\sum_{i=1}^N \lambda_i(\mu_i+s_i))}(\sum_{i=1}^N t_i^{(1)})$ and variance $\frac{1}{3}\left(\frac{1-\lambda_1(\mu_1+s_1)}{1-\sum_{i=1}^N \lambda_i(\mu_i+s_i)}\right)^2 (\sum_{i=1}^N t_i^{(1)})^2$.

5 Numerical results and application

5.1 Compare two sorting policies

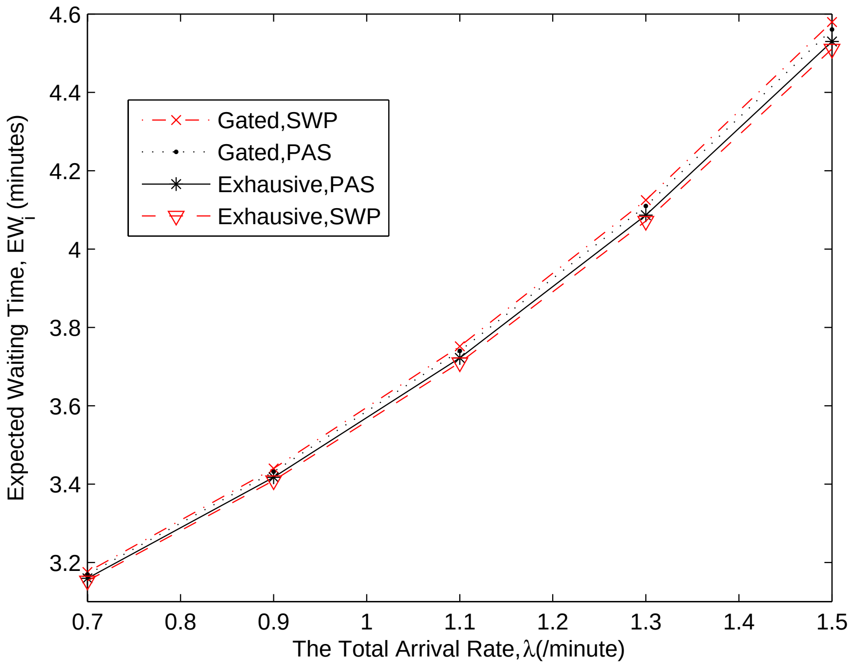

Figure 2: Comparing sorting policies.

We compare "sort-while-pick" policies and "pick-and-sort". In "sort-while-pick" policies, the sorting time becomes a part of service time in a polling system. In "pick-and-sort" policies, the sorting time becomes a part of setup time in a polling system. To make this comparison relatively fair, we set τ_s, such that expected sorting time in "pick-and-sort" policies equals to the total expected additional sorting time in "sort-while-pick" processes. Let $E[C]$ be the expected cycle time of the order picker, i.e., the time between two consecutive arrivals of the picker at the same storage position, and the cycle time is $E[C] = t/(1-\rho)$ (see [27]). The expected arrival items during one cycle time is $\sum_{i=1}^{N} \lambda_i t/(1-\rho)$. Total sorting time for "pick-and-sort" is set as $\tau_s = s\sum_{i=1}^{N} \lambda_i t/(1-\rho)$, which is $\tau_s = s\lambda(\tau_t+\tau_s)/(1-\rho)$. Therefore we can calculate the total sorting time for "pick-and-sort" as $\tau_s = \frac{s\lambda\tau_t}{1-\rho-s\lambda}$.

We use a set of parameters within the scope in existing literature [18] since both papers consider manual order picking with online arrivals and two different sorting policies. In the experiment, the travel time is 300 seconds, the position number is 60, the unit picking time is 8 seconds per order line, and the unit sorting time is 10 seconds per order line. We consider a total arrival rate from 0.7 to 1.5 per minute.

Table 3: Sensitivity analysis

Group	Index	Exhaustive, PAS	Exhaustive, SWP	Gated, PAS	Gated, SWP
1, Size	1 ($\tau_t = 4$)	3.6243	3.6091	3.6485	3.6636
($\lambda = 1.5$	2 ($\tau_t = 5$)	4.5303	4.5114	4.5606	4.5795
$s = 0.167$	3 ($\tau_t = 6$)	5.4364	5.4136	5.4727	5.4955
$\mu = 0.133$)	4 ($\tau_t = 7$)	6.3425	6.3159	6.3848	6.4114
2, λ	5 ($\lambda = 1.6$)	4.7906	4.7692	4.8247	4.8462
($\tau_t = 5$	6 ($\lambda = 1.7$)	5.0828	5.0587	5.1213	5.1454
$s = 0.167$	7 ($\lambda = 1.8$)	5.4131	5.3859	5.4565	5.4837
$\mu = 0.133$)	8 ($\lambda = 1.9$)	5.7895	5.7587	5.8384	5.8692
3, s	9 ($s = 0.12$)	4.0156	4.0035	4.0424	4.0545
($\tau_t = 5$	10 ($s = 0.14$)	4.2196	4.2048	4.2478	4.2626
$\lambda = 1.5$	11 ($s = 0.16$)	4.4455	4.4276	4.4751	4.4930
$\mu = 0.133$)	12 ($s = 0.18$)	4.6969	4.6757	4.7282	4.7494
4, μ	13 ($\mu = 0.13$)	4.4939	4.4751	4.5232	4.542
($\tau_t = 5$	14 ($\mu = 0.15$)	4.7486	4.7287	4.7843	4.8042
$\lambda = 1.5$	15 ($\mu = 0.17$)	5.0341	5.0130	5.0771	5.0982
$s = 0.167$)	16 ($\mu = 0.19$)	5.3566	5.3341	5.4077	5.4302

From Figure 2, the performance difference between "pick-and-sort" and "sort-while-pick" is not very huge, which confirm our calculate method for the total sorting time τ_s for "pick-and-sort" is reasonable. The experiments also confirm that the performance of exhaustive order picking policy is better. Moreover, Figure 2 shows important relation between sorting policies and picking policies. (1) For exhaustive order picking policy, "sort-while-pick" policy is better for all tested cases. (2) For gated order picking policy, "pick-and-sort" policy is better for all tested cases. It fits to operation protocols. For exhaustive policy, during the sorting at a storage position, the arrival order lines can be picked up in time. But with a gated policy, during the sorting at a storage position, the arrival order lines are not allowed to be picked up in time. This experiment provides new insights on allocating sorting time. To achieve maximal real-time response performance, a "sort-while-pick" policy with an exhaustive picking policy may be better than other policies.

To test the sensitivity of the result, we generate different scenarios in table 3. In group 1, we change different warehouse sizes and thereby different traveling time. We respectively experimented with different arrival rates, picking time, and sorting time in group 2, 3, and 4. We also test asymmetric scenarios, where the arrival rates are different at different storage positions since our model can admit this general cases. The results are consistent: an exhaustive order picking policy combined with "sort-while-pick" policy is better for all tested cases.

5.2 The second moment performance measurement

The ability to measure, understand, and manage variability is critical to effective operations management (see [28]). Our method can provide high moment performance

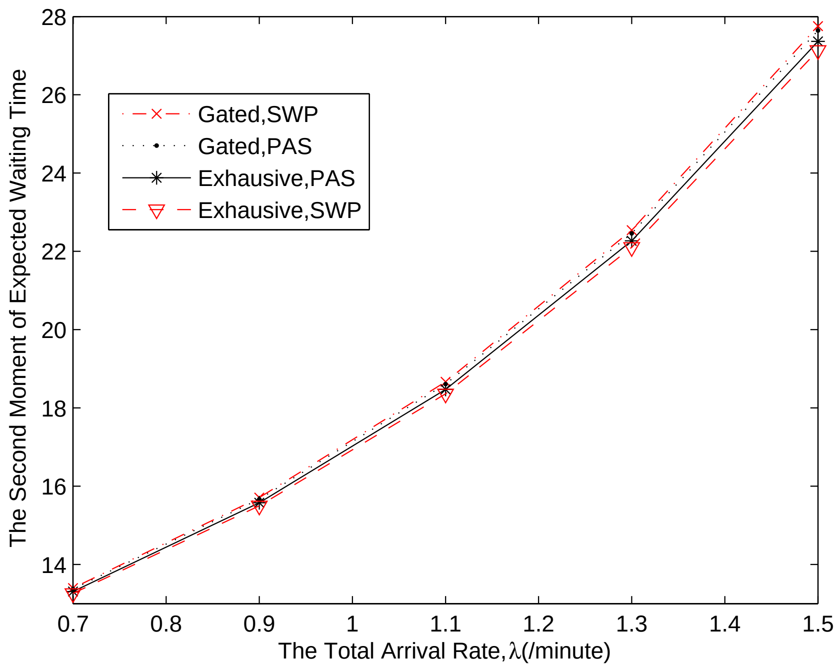

Figure 3: The second moment performance measure.

measurement to a RPSS system.

In the Figure 3, we show the second moment of expected waiting time in five systems with the same parameters in section 5.2. From the figure, the second-moment performance of exhaustive order picking policy is better than that of gated order picking in both sorting policies. We find the second-moment performance of "sort-while-pick policy" is better when using exhaustive policy, which confirm the results in terms of the mean values.

5.3 Compare RPSS systems with batch picking

This section compares RPSS systems and traditional batch picking systems with the same parameters in section 5.2. By simulation, we calculate the performance measure of a traditional batch picking system with optimal batch sizes, for the same arrival rate and service capacity. For the batch picking systems, we use an enumeration method to obtain the optimal batch size (see Le-Duc and De Koster (2007)). The optimal batch size differs for different arrival rates and varies between 4 and 11. For this system,we use an FCFS batching policy, an S-shape routing policy and random storage. We assume single-line orders. For batch picking, we assume a batch sorting, i.e., "pick-and-sort" protocol. In traditional batch picking, the pick locations during a batch pick cycle are given and fixed, and pick information is not updated during the cycle. We simulate 20 540 randomly generated order lines and compute the average order waiting time.

The performance comparison of a polling system and a batch picking system is presented in Figure 4. We can observe that the expected order waiting time of a batch picking system is longer than RPSS systems.

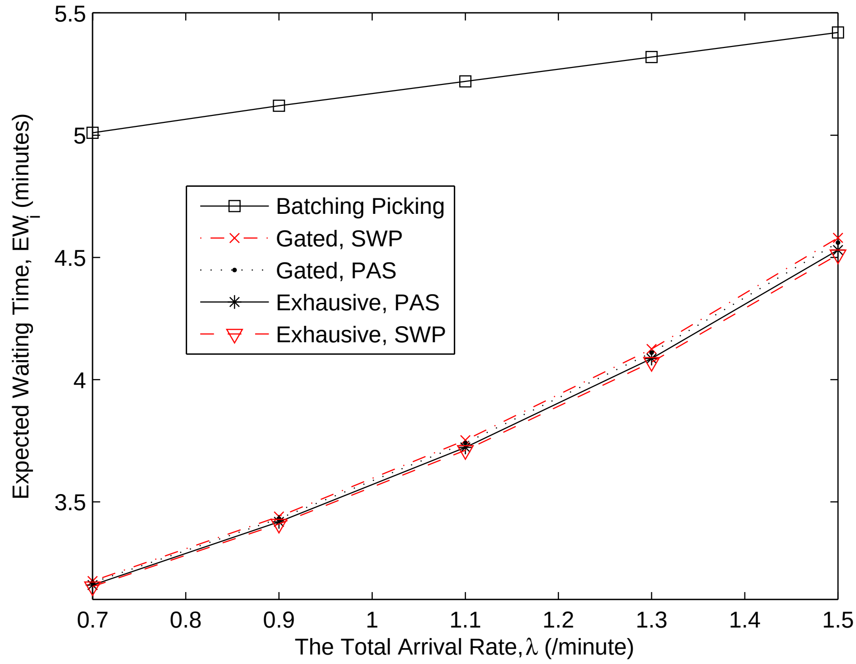

Figure 4: The comparison of RSPP systems with a batch picking system.

6 Concluding remarks

This paper researches the application of real-time operational strategy on order fulfillment, the most expensive and critical operation for e-commerce companies. Particularly, we identify an order picking and sorting system for e-commerce distribution centers. In this paper, we use polling models to describe and analyze such operations in warehouses.

We find approximate closed-form expressions for the order line waiting times in systems with two different order picking protocols and two different sorting protocols and are able to present the second moment measure for the system performance. We provide a fast algorithm to satisfy the requirement on the running time in an environment of real-time operations. This study provides new insights into allocating sorting time in e-commerce distribution centers.

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