

# **ANALYSIS OF PICKER BLOCKING IN NARROW-AISLE BATCH PICKING**

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## **Abstract**

This study analyzes impacts by batch picking on picker blocking in narrow-aisle order picking, and determines appropriate batch formations for a better order picking throughput. We present multiple-pick analytical models to offer insights about picker blocking in batch picking. Several simulation studies over a variety of batching situations scrutinize order batching situations which give throughput benefits in narrow-aisle configurations by satisfying the analytical results. Our results highlight three findings for narrow-aisle batch picking processes: 1) variation of pick probability in batch picking is inevitable and is a primary driver of picker blocking; 2) a near-optimal distance-based batching algorithm can experience less picker blocking than expected, because it reduces both the number of aisles visited and the variation in the number of picks per aisle; and 3) the sorting strategy itself (i.e., pick-then-sort or sort-while-pick) causes varying amounts of congestion.

## **1. Introduction**

Picking orders in batches is often favored when customers' demands create a large number of small orders. Yet, when more pickers are included in the picking process, order picking performance is likely to decline due to significant picker blocking. In particular, narrow-aisle picking environments, which are attractive for their storage capability can produce picker blocking, even though one-way traversal routing is used to mitigate congestion [1]. Accordingly, the order fulfillment time can lengthen and operational costs increase. In practice, the effects of batch formation on picker blocking vary according to the batching algorithm, sorting strategy, and storage policy. In other words, the process of assigning orders to particular batches can give management the additional flexibility needed to reduce picker blocking.

To date, researchers do not fully understand the relationship between picker blocking and batch formation. For example, in a preliminary study on a parallel-aisle system, we found that a batch formation generated by a batching strategy where the number of orders determined the batch size experienced less picker blocking than a batch formation generated by a strategy constrained by the number of items, *even though* both formations exhibited a similar pick density. This observation cannot be explained by available analytical studies [3, 8].

A principle of batch picking is to have pickers gather items that are closely located within the storage space when feasible. Basically, a batch has a higher pick density compared to single order, which leads to higher picker utilizations. Two studies [1-2] consider a model under a single-pick assumption defined as a situation in which only a single product type is picked at a particular pick face. However, in batch picking, the probability of needing to pick more than one product type at a particular pick face increases. Thus, multiple-pick models which consider repeated picks at a particular pick face can be useful.

This paper develops analytical models of picker blocking considering multiple-picks in narrow-aisle configurations. We compare our models to the single-pick models by Gue *et al.* [3] to develop a deeper understanding of picker blocking in narrow-aisle batch picking. We also conduct simulation studies over different batch picking situations to support the findings from our analytical models. More importantly, we relate characteristics of the picking environment and order requirements to determine appropriate batching strategies.

This paper is organized as follows. Section 2 defines picking blocking in a narrow-aisle batch picking. In Section 3 we derive a two-picker multiple-pick blocking model and apply it to two extreme cases (pickers' speed is very fast or slow). Relevant insights about the differences are discussed. Section 4 examines the relationship between analytical models and batching situations. Section 5 summarizes the findings and offers suggestions for future research.

## **2. Problem definition**

### **2.1 Batch picking in narrow-aisle picking systems**

In narrow-aisle picking systems, pickers circumnavigate one-way aisles to retrieve items from shelves and place them in a cart as shown in Figure 1. When an aisle includes no items assigned to the picker, the aisle can be skipped to shorten the travel distance if the unidirectional characteristic of the aisles can still be maintained. In practice, the order size is relatively small compared to the cart capacity; thus, a way to improve the order picking throughput is to consolidate multiple orders to a single trip ("batch picking"). Clearly, total retrieval time is reduced when pickers can collect multiple orders in the same trip. Orders cannot be split over multiple batches and batch size is determined by the cart's carrying capacity. In a narrow-aisle picking system, picker blocking can occur when multiple pickers traverse a pick area while maintaining a no-passing restriction. An

upstream picker cannot pass a downstream picker because of the aisle's width. When a downstream picker is picking, *picker blocking* arises because the upstream picker cannot pass.

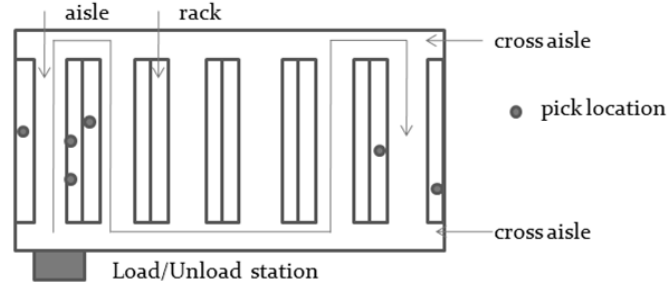


Figure 1: A narrow-aisle system and a routing example (modified from Gademann and Van de Velde [2]).

## 2.2 Throughput model

Order picking systems are often characterized by *ratio of time spent to pick to time spent at a stop*. It is expected that time spent at a stop includes only pick time and walk time. However, in general, picker blocking is inevitable and impacts the throughput. Gue *et al.* [3] introduced the throughput model for an order picking system with  $k$  pickers in a single-pick situation. To reflect a multiple-pick situation, we generalize their model as Equation (1). When each picker is blocked with a fraction of the time,  $b(k)$ , where  $0 \leq b(k) \leq 1$ , the throughput is:

$$\lambda(k) = k \cdot \left[ \frac{E[pt]}{E[pt]t_p + t_w} \right] (1 - b(k)) \quad (1)$$

where  $E[pt]$  stands for the expected number of picks at a stop. The time to pick ( $t_p$ ) represents the average time the picker is stopped and includes the time spent picking items. The time to walk ( $t_w$ ) indicates the average time to walk past a pick face (location). In a single-pick model,  $E[pt]$  is equal to  $p$  [3], but a multiple-pick model is also affected by the number of expected picks at a particular pick face as described in Parikh and Meller [5].

## 2.3 A circular order picking aisle model

To simplify the analysis of the picker blocking phenomena in a narrow-aisle picking system, a parallel-aisle system is often modeled as a circular order picking aisle as shown in Figure 2. In developing the blocking models, we assume the following: 1) the circular order picking aisle consists of  $n$  pick faces; 2) two pickers perform the order picking; 3) they take a one-way traversal route, meaning that they travel through that aisle in only one direction (in the circular model this implies that they move only in a clockwise direction); 4) pick time is constant regardless of the pick face characteristics such as shelf height; 5) at a pick face, pickers pick with a probability  $p$ ;  $q$  denotes  $1-p$ , the probability of walk at a pick-face; after completion of a pick, pickers repeat the same procedure; 6) a

picker can only be picking, walking, or standing idle due to blocking; 7) the pick time and the walk time between two pick faces are deterministic, and termed as  $t_p$  and  $t_w$ .

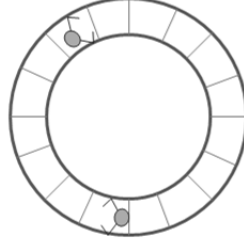


Figure 2: A circular order picking aisle [3].

As a performance measurement, we obtain the percentage of time blocked, denoted as  $b_{pt:wt}^m(k)$ .  $m$  stands for a multiple-pick situation and  $pt:wt$  represents the walk:pick time ratio. In the case of a single-pick situation ( $s$ ), Skufca [8] previously derived the analytical model for  $b_{1:0}^s(k)$ . Gue *et al.* [3] studied  $b_{1:1}^s(2)$  and  $b_{1:0}^s(2)$  analytically, and generalized other cases (e.g.,  $b_{1:0.5}^s(2)$ ,  $b_{1:0.25}^s(2)$ , ...,  $b_{1:1}^s(10)$ ) using simulation models.

## 2.4 Scope of study

This paper aims to develop analytical models for  $b_{1:1}^m(2)$  and  $b_{1:0}^m(2)$ . Further, for a better understanding of picker blocking and batch picking and their relationships to other aspects of the warehouse, we conduct an extended simulation study considering batching algorithms, sorting strategies, and picking system configuration (i.e., the number of aisles) in a parallel-aisle picking system.

## 3. Picker blocking models for multiple-picks

We first build analytical models for two order pickers who conduct a retrieval operation in a parallel-aisle picking system using the circular aisle characterization to develop a general understanding, and then conduct a comparison study between single-pick and multiple-pick models to identify the significance of picker blocking.

Our analytical study considers two extreme cases which do not exist in practice, but can bound all actual situations and provide an excellent understanding of picker blocking: 1) walk speed is equal to unit pick time per pick face (pick:walk time = 1:1), and 2) walk speed is infinite (pick:walk time = 1:0). Our analytical model utilizes a Markov property in determining distances between two pickers; see also [3, 5, 8].

### 3.1 Pick:walk time = 1:1

Let  $D_t$  denote the distance between picker 1 and picker 2 at time  $t$ . Given the pick:walk time ratio as 1:1, the distance  $d$  can be expressed as

$$(n+(\text{picker 1 position})-(\text{picker 2 position})) \bmod n \quad (2)$$

and range from 1 to  $n-1$ . A Markov chain is introduced by defining state  $S_t = 0$  (*block*) representing picker 1 blocking picker 2, state  $S_t = n$  representing picker 2 blocking picker 1, and states  $[1, 2, \dots, n-1]$  are given by  $S_t = D_t$ . In other words, there are two blocking

states and  $n-1$  distance-related states. Shortly, all states can be described as [blocked, 1, 2, ...,  $n-1$ , blocked].

These states allow us to distinguish four transition cases: 1) transition between unblocked states; 2) transition from an unblocked state to a blocked state; 3) transition from a blocked state to an unblocked state; and 4) transition between blocked states.

**1) Transition probabilities between unblocked states**

If both pickers pick ( $p*p$ ) or walk ( $q*q$ ), the current distance ( $D_t$ ) does not change at  $t+1$ . However, when picker 1 picks while picker 2 walks ( $p*q$ ), the distance decreases by 1. When picker 1 walks while picker 2 picks ( $q*p$ ), the distance increases by 1.

**2) Transition probabilities from an unblocked state to a blocked state**

When the distance from picker 1 to picker 2 is 1, a blocked state can arise if picker 1 picks (with probability  $p$ ), and picker 2 walks (with probability  $q$ ). Vice versa, when the distance from picker 1 to picker 2 is  $n-1$ , the current state becomes a blocked state if picker 1 walks (with probability  $q$ ) and picker 2 picks (with probability  $p$ ).

**3) Transition probabilities from a blocked state to an unblocked state**

If picker 1 is blocked by picker 2, picker 1 must wait for picker 2 to walk (with probability  $q$ ) to exit a blocked state. Vice versa, when picker 2 is blocked by picker 1, picker 2 must wait for picker 1 to walk (with probability  $q$ ).

**4) Transition probabilities between blocked states**

When the current state is blocked, a pick can occur with probability  $p$ . Blocking status remains, i.e., a blocked state transits to a blocked state with probability  $p$ .

In sum, when multiple picks are allowed, the transition probability can be described as illustrated in Figure 3.

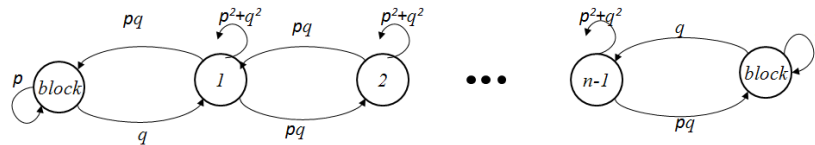


Figure 3: State space and transitions for the Markov chain model when picking time equals travel time.

The Markov chain model in Figure 3 does not include substates of picking or walking as in Gue *et al.* [3]. Thus, the transition matrix is more condensed. The resulting transition matrix which has dimensions  $(n+1) \times (n+1)$  is:

$$A = \begin{bmatrix} p & q & 0 & \cdots & 0 & 0 & 0 \\ pq & p^2 + q^2 & pq & \cdots & 0 & 0 & 0 \\ 0 & pq & p^2 + q^2 & \ddots & \ddots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & \ddots & \ddots & p^2 + q^2 & pq & 0 \\ 0 & 0 & 0 & \cdots & pq & p^2 + q^2 & pq \\ 0 & 0 & 0 & \cdots & 0 & q & p \end{bmatrix}$$

### **Stationary distribution**

The following  $v$  is obtained which satisfies  $vA = v$ .

$$v = \left[ 1, \frac{1}{p}, \dots, \frac{1}{p}, 1 \right]$$

The stationary density using  $\|v\|$  is scaled to obtain a stationary probability. From  $v$  above, this implies:

$$\|v\| = 2 \cdot 1 + (n-1) \frac{1}{p} = 2 + \frac{n-1}{p}$$

The blocking probability of one picker at one blocked state is

$$b_{i1}^m(2) = \frac{v_{1*}}{\|v\|} = \frac{1}{2 + \frac{n-1}{p}} = \frac{p}{2p + n - 1} \quad (3)$$

### **3.2 Pick:walk time = 1:0**

The infinite speed assumptions allow for transitions to multiple states in our Markov chain model. Thus, the probability that a picker moves distance  $x$  is approximated, and then a probability function for the distance  $y$ , characterizing the change in the distance between the two pickers, is estimated.

Let random variables  $X_t^1$  and  $X_t^2$  represent the desired number of locations moved in time  $t$  by pickers 1 and 2, respectively. If a picker picks more than one pick at a pick face, the distribution of the location follows over the infinite sample space of the random variables of the desired number of locations:

$$f(x) = q^x p \quad \text{for } x = 0, 1, 2, \dots \quad (4)$$

$Y_t = X_t^1 - X_t^2$  denotes the change in distance between the two pickers when passing is not allowed. As described in Parikh and Meller [5], the probability density function of  $Y_t$  ( $g(y)$ ) becomes

$$g(y) = \frac{pq^{|y|}}{1+q} \quad \text{for } -\infty < y < \infty \quad (5)$$

Suppose the distance at the previous state is  $D_{t-1} = r$ . The actual change in distance is bounded by the physical blocking phenomenon and the amount of the change is limited by  $r$ . Two transition cases are defined: 1) transition to unblocked states; and 2) transition to blocked states.

### 1) Transition probabilities to unblocked states

In this case, the distribution function (5) is used directly. Note that  $r$  is 0 or  $n$  when a picker is blocked. Thus, the change with given  $r$  is available within 1 to  $n-1$  to prevent the change from outpacing unblocked states:

$$P(Y_t = y) = g(y) = \frac{pq^{|y|}}{1+q} \quad \text{for } 1-r < y < n-1-r, r=0, \dots, n. \quad (6)$$

### 2) Transition probabilities to blocked states

The next step is calculating the probability of events with blocking. To obtain this probability, we need to cumulate all cases above the limits (0 or  $n$ ). We note that there will be blocking at state 0 if  $Y_t \leq -r$ .  $g(y)$  is symmetric and can be calculated as:

$$P(Y_t \geq r) = \sum_{y=r}^{\infty} \frac{pq^{|y|}}{1+q} = \frac{p}{1+q} q^r \frac{1}{1-q} = \frac{q^r}{1+q}$$

$$P(Y_t \geq n-r) = \frac{q^{n-r}}{1+q}$$

The result forms the transition matrix:

$$A = \begin{bmatrix} \frac{1}{1+q} & \frac{pq}{1+q} & \frac{pq^2}{1+q} & \dots & \frac{pq^{n-2}}{1+q} & \frac{pq^{n-1}}{1+q} & \frac{q^n}{1+q} \\ \frac{q}{1+q} & \frac{p}{1+q} & \frac{pq}{1+q} & \dots & \frac{pq^{n-3}}{1+q} & \frac{pq^{n-2}}{1+q} & \frac{q^{n-1}}{1+q} \\ \frac{q^2}{1+q} & \frac{pq}{1+q} & \ddots & \ddots & \ddots & \frac{pq^{n-3}}{1+q} & \frac{q^{n-2}}{1+q} \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ \frac{q^{n-2}}{1+q} & \frac{pq^{n-3}}{1+q} & \ddots & \ddots & \ddots & \frac{pq}{1+q} & \frac{q^2}{1+q} \\ \frac{q^{n-1}}{1+q} & \frac{p}{1+q} & \frac{pq^{n-2}}{1+q} & \dots & \frac{pq}{1+q} & \frac{p}{1+q} & \frac{q}{1+q} \\ \frac{q^n}{1+q} & \frac{pq^{n-1}}{1+q} & \frac{pq^{n-2}}{1+q} & \dots & \frac{pq^2}{1+q} & \frac{pq}{1+q} & \frac{1}{1+q} \end{bmatrix}$$

### Stationary distribution

To identify a stationary distribution, a  $v$  which satisfies  $vA=v$  is identified as:

$$v = [1, p, \dots, p, 1]$$

We can scale the stationary density using  $\|v\|=2+(n-1)p$ . The blocking probability of a picker at one blocked state is:

$$b_{t;0}^m(2) = \frac{1}{2+(n-1)p} \quad (7)$$

### 3.3 Multiple-pick models versus single-pick models

Figure 4 compares a multiple-pick model ( $m$ ) and a single-pick model ( $s$ ) over two different numbers of pick faces (20 and 50 pick faces). Here, the x-axis is the average number of picks, not pick density. The multiple-pick models consistently experience a higher productivity loss by picker blocking, even though the number of picks is identical.

In general, the multiple-pick model can have more variation of picks than the single-pick model. The variations affect picker blocking.

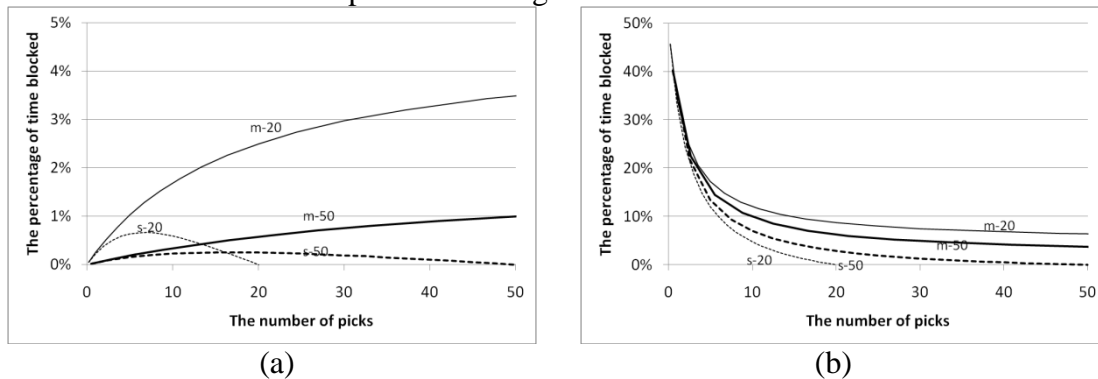


Figure 4: The percentage of time that pickers are blocked when two pickers work with: (a) pick:walk time =1:1 and (b) pick:walk time =1:0.

#### 4. Simulation study

The two analytical models are based upon four assumptions: 1) extreme pick:walk time ratio; 2) a Markov property in distance; 3) the circular approximation to a parallel aisle order picking area, and 4) no batching algorithm effect. In practice, pickers are not extremely fast or slow. If the pick time is 1, most practical speeds for walking are in the range of 0.05 to 1 [3]. For example, our literature review found a fast speed would have a pick to walk ration of 1:0.1 [7] and a slow speed would have a ratio of 1:0.2 [9]. Another difficulty analyzing picker blocking in real picking situations arises due to the multiple-aisle characteristic and the impacts of routing. Last, the analytical models do not consider pick formation modification by order batching. In other words, the batch formation after employing a batch algorithm differs from a batch formation generated by the analytical model.

##### 4.1 Fractional walk speed and non-Markov property in distance

Two restrictions (assumptions 1 and 2) are relaxed. As walk speed, the ratio of pick:walk time = 1:0.2 is assumed. Relaxation of a Markov property in distance influences the variation of the batch size across batches. Several models to determine the batch size are considered: a single-pick model as the single-pick analytical model (Gue *et al.* [1]), a multiple-pick model generated with the restrictions in the multiple-pick analytical model (described above), a fixed-size model where the number of picks in a trip is constant and multiple-picks are allowed, and a uniform-size model where the number of picks in a trip follows a discrete uniform distribution [mean/2, mean \*3/2] with allowance of multiple-picks.

Figure 5(a) depicts the relationship between the percentage of time blocked and “the number of picks” for different assumptions regarding the distribution of items and Figure 5(b) illustrates the relationship between “the number of picks” and the variation of “the number of picks” for different assumptions regarding the distribution of items. A high



variation in the number of picks per trip results in more severe picker blocking, and conversely, even if the number of picks in a trip is large, i.e., pick density is high and multiple-picks are allowed, if the variation in the number of picks is low there is less picker blocking (see the fixed-size model).

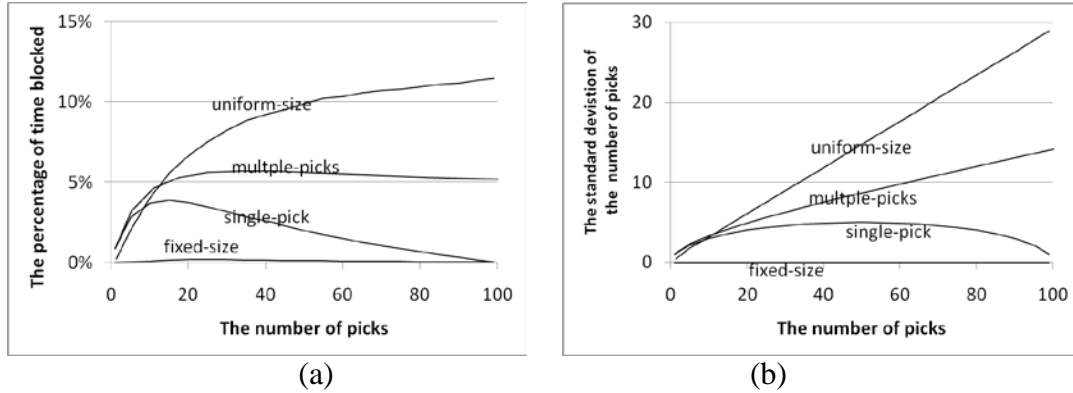


Figure 5. Simulation results over different workload distributions (the number of pickers = 5, the number of pick faces = 100, and pick:walk time = 1:0.2) : (a) the percentage of time blocked; and (b) the standard deviation of the number of picks (workload).

## 4.2 Batch picking in pparallel-aisle picking systems

A simulation study in a parallel-aisle order picking system with batching algorithms is conducted to observe the effects of the batching algorithms, sorting strategies, and picking system configurations. It is evident that picker blocking is observed in large-scale order picking. Thus, we survey algorithms which can manage large-scale order batching situations. From the available literature, we consider the following algorithms:

- FCFS: batches are conducted from orders in a first-come first-served manner.
- Seed: the seed algorithm developed in De Koster *et al.* [1]. This algorithm is characterized by excellent computational efficiency, but a relatively poor solution quality compared to RBP.
- RBP: the heuristic route-selection-based batching algorithm [4]. The route-selection batching procedure (RBP) is a near-optimal batching algorithm developed by Hong *et al.* [4].

A sorting strategy relates to batching algorithms while forming a batch size. The sort-while-pick strategy (SWP) and the pick-then-sort strategy (PTS) are considered. Products are often stored in a warehouse to minimize the effort in retrieving the items. We assume that items are stored in random locations in the warehouse.

Two picking system layouts are considered: a two-aisle order picking situation and a ten-aisle order picking situation. A two-aisle order picking environment is defined as: the number of aisles = two; the number of pick faces per aisle = 200 pick faces; the number of orders in a time window = 540 orders; eight time windows; pick:walk time ratio =

1:0.2; five pickers; setup time per batch = 120; and cart capacity = ten orders or 20 items. A ten-aisle environment is defined as above except: ten aisles and 20 pick faces per aisle.

Figure 6 depicts the comparison of the total retrieval time. RBP is relatively robust, while the seed algorithm can produce very poor results due to heavy congestion. Figure 6 emphasizes the importance of picker blocking and selecting a batching algorithm that reduces travel distance and does not create excessive picker blocking.

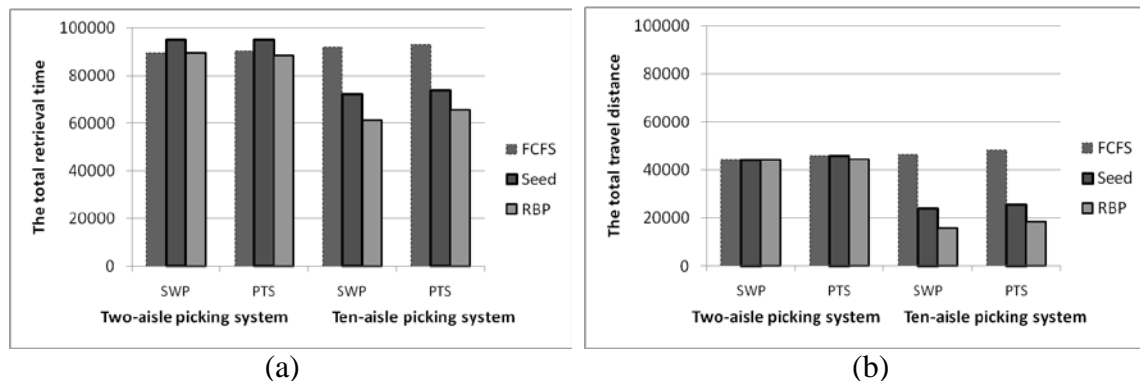


Figure 6: The performance comparison via a simulation study: (a) the total retrieval time; and (b) the total travel distance.

Figure 7 details the throughput loss by the time blocked and the variation in the number of picks per aisle. The seed algorithm creates heavy congestion compared to FCFS. However, the RBP solution exhibits less congestion ratio compared to the seed algorithm even though its solution quality approaches to near-optimal in the travel distance. Interestingly, the standard deviation of RBP is less than the standard deviation of the seed algorithm. Intuitively, an improved distance-based batching algorithm could encounter more congestion. However, RBP reduces congestion due to large reductions in the distance traveled, and relatively reasonable variation of picks per aisle.

Sorting strategy impacts picker blocking when combined with RBP. In the two-aisle picking system, only single route is available under the traversal routing method. The pick-then-sort strategy determines the batch size by the number of picks. As batches are more optimally consolidated, their size becomes identical and the batching algorithm can lessen the variation of the number of picks across batches. Therefore, picker blocking decreases.

The ten-aisle picking system faces a different situation as the number of aisles visited across batches becomes diverse. When the sorting operation is separated from the order picking operation (i.e., pick-then-sort strategy), the routing options used will impact the variation of the number picks in unit distance (i.e., number of picks per aisle) across batches. The variance of the number of picks per aisle increases depending on the number of routing options. In the SWP strategy, less variation of picks per aisle can be achieved while obtaining a high quality solution. The sort-while-pick strategy constrains each batch to have the same number of orders, not number of items. The expected number of

picks of a batch will typically be proportional to the length of route, i.e., the number of aisles visited, as batches are packed more optimally. Thus, compared to the PTS strategy, this characteristic can produce less variation of the number of picks per aisle, which reduces picker blocking.

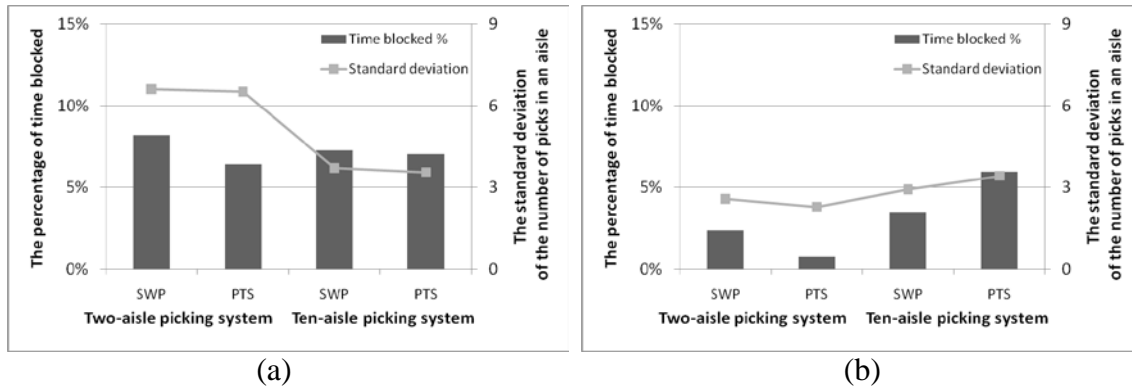


Figure 7: Analysis of the percentage of time blocked and the variation of the number of picks over: (a) the seed algorithm; and (b) the RBP algorithm.

## 5. Conclusion and further study

This research has provided a new understanding of picker blocking in a narrow-aisle batching picking situation and has scrutinized the relationship between picker blocking and order batching using analytical models and simulation studies. Picker blocking in the multiple-pick model did not decrease to 0 time blocked as pick density increased, as shown in the more restrictive single-pick model. The comparison between the single-pick model and the multiple-pick model shows that a high variation in the number of picks in an aisle can cause serious picker blocking regardless of the number of picks in an aisle.

A simulation was conducted for several practical situations: batching algorithm, sorting strategy, and different picking system configuration. Most importantly, simulation results reveal that a near-optimal distance-based batch algorithm (RBP) creates very little picker blocking. Furthermore, the sorting strategy affects the variation of the number of picks in an aisle, thus making certain sorting strategies (sort-while-pick) more effective in certain layouts (a large number of aisles).

Our current research involves identifying additional observations from the two analytical models. For example, two models show unique convergences over pick density. We suggest that explicit consideration of picker blocking in a batching model is needed. Explicitly modelling and controlling picker blocking has the potential for significant improvements.

## **Addendum**

After our paper was written, Parikh and Meller's paper [6] appeared in the literature. Our paper overlaps with their paper in some areas, but addresses them from a different perspective and presents similar but not identical results.

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