ANALYSIS OF MATERIAL HANDLING SYSTEMS BASED ON DISCRETE TIME DESIGN MODULES

Furmans, Kai
Özden, Eda
Stoll, Judith
Epp, Martin
Karlsruhe Institute of Technology

Schmidt, Thorsten
Meinhardt, Ingolf
Schulze, Frank
Technical University of Dresden

Abstract

In this paper, we present a method for the performance evaluation of material handling systems in an early planning stage. We are mapping typical material handling elements onto stochastic analytical models. Our intention is to build a toolbox of material handling modules that allows a fast analysis of typical continuous conveying systems. The tool should enable the identification of bottlenecks and the computation of lead time distributions for intra-logistic systems.

1 Introduction

During the planning phase of a material handling system it is necessary to assure, that the design fulfills the operational requirements. A first check can be done by looking at the utilization of the individual elements of the design. However, this does only guarantee that the required throughput can be achieved over time - and it only works if the system does not suffer from negative feedback which results for instance from blocking or too long cycle times of vehicles in a closed system. Therefore, queuing models have been used as a method to achieve a fast and more accurate way of determining performance figures during the planning phase.

Since 2004, work has been done in Karlsruhe on queuing models in discrete time, which can be combined using operational data for processing times and the output of preceding elements as input for succeeding model elements of the material handling system. Several basic elements have been treated analytically which can now be combined (Stochastic Finite Elements, see publications of Schleyer, Özden, Matzka/Stoll, and Furmans between 2004 and 2011). The advantage of these models lies in the fact, that they allow the computation of not only averages but also the distributions of e.g. waiting times. This type of models requires good program support for the computations and modeling, since the amount of data used and created as well as the computation itself cannot be handled manually. Scientists as well as practitioners will only be able to use these mathematically more advanced methods, if there is computer supported modeling and computation available.

At TU Dresden, a research effort has been made during the last years, which was also focused on modeling large material handling systems by means of queuing networks, with an emphasis on computer modeling. The underlying analytical model was based on continuous time models. First experiences with practical applications have already been made in Dresden. Now the efforts of both research groups have been combined, in order to achieve the following targets:

- A suitable modeling methodology should be defined by mapping typical material handling elements onto stochastic analytical models
- By mapping models of large material handling systems onto this modeling methodology, model elements are identified, where a computation algorithm for the performance figures is not known yet
- These missing algorithms should be developed
- By comparing the results of the analytical evaluation with simulation results of the same model, systematic difference should be identified and correction factors should be determined

We want to show and discuss our means of modeling and the capability of the performance calculation tool as well as its integration within the modeling GUI. This will hopefully lead to a further discussion within the community, how to open the system for other contributors. For many of the currently as necessary identified single elements, exact algorithms or good approximations are known through the past work. The challenge lies now mostly in modeling control strategies and calculating the impact, these control strategies have on throughput and throughput (lead) times.

The paper is organized as follows: First, we will present the modular modeling technique in section 1, that is based on single design modules. These single design modules are based on discrete time calculation methods. We will briefly present the idea of discrete time modeling and its advantages in section 2. Section 3 gives an overview of the results that can be obtained using the modular modeling technique that is implemented in a performance calculation tool with a modeling GUI. In section 4, we identify the missing algorithms that should be developed in order to be able to analyze typical material handling systems. For some of the missing algorithms we already found a solution. This will be demonstrated by modeling and analyzing a

4-way-crossing of roller-conveyors under the time-limited service control strategy (see section 5).

2 Model design of a modular material flow system

For the methodological support of the early planning phase, the idea of a "design module" arises: in the same way as a material flow system is structured in functional areas and assemblies, the calculation of overall system behavior is also based on calculations of its system components [5]. A material flow system is, therefore, regarded as a node-edge model [8]. The nodes represent the components of the material flow system. Typical operations in intra-logistic systems are e.g. checking, packaging or manual handling processes, but also basic operations on the material flow, such as the branching and merging of goods, quantity changes by collection and distribution processes, as well as sorting and buffering of goods.

As the edges in technical systems (e.g. conveyor belts) transfer streams of goods, they transfer, in the mathematical model, information on the material flow in form of discrete probability distributions to the subsequent nodes. In addition, they can receive other information on such as transport and waiting time distributions for a later evaluation.

For the modeling process, a multi-level hierarchy is provided, which enables a step-by-step detailing of the required hierarchy level (e.g. storage area, picking area or similar) down to the individual design module. A design module is a model of a concrete material flow component (e.g. storage retrieving machine, roller lift table, etc.), which is described based on its functional specification (e.g. steadily / uneven, different service strategies), and with component-specific parameters. In addition, intensity and variability of the incoming transport stream are provided through connections from other design modules. Thus, the utilization can be assessed: a high utilization (busy time) of the component can result in queues (and thus waiting time) in front of the module. By means of computational models assigned to the design module, the flow behavior after the element is quantified in form of a discrete distribution. This distribution is then used as the arrival distribution for the subsequent design module (see figure 1).

3 Discrete time methods for the analysis of single design modules

The analysis tool is based on discrete-time analytical models. Whereas in continuous-time calculation methods (e.g. classical queuing models) characteristic values are calculated only on the basis of means and variances, in discrete-time modeling all input and output variables are described with discrete probability distributions. Thereby

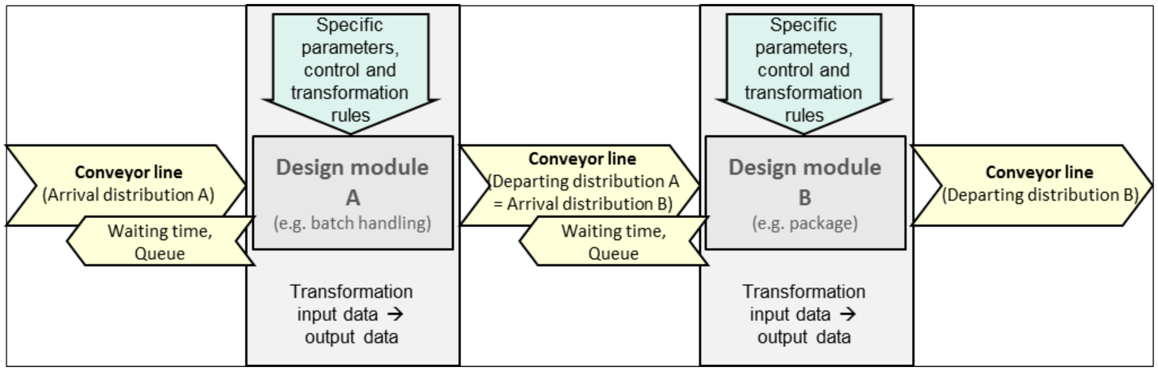

Figure 1: Results of modular calculations are mapped to the arriving and departing conveyor lines

the discretization of the time is not a limitation. On the contrary, operational times in material handling systems exist very often in form of discrete values, such as a transfer carriage, a storage retrieval machine or a turntable.

Figure 2 shows this on the example of a carriage with three branch directions and the associated probability distribution of the service time. In the given example, the conveying time takes only 3 distinct discrete values. The exact, possibly empirically obtained values can be used directly as input to the model and do not have to be approximated by theoretical distributions. Also for the determination of key figures,

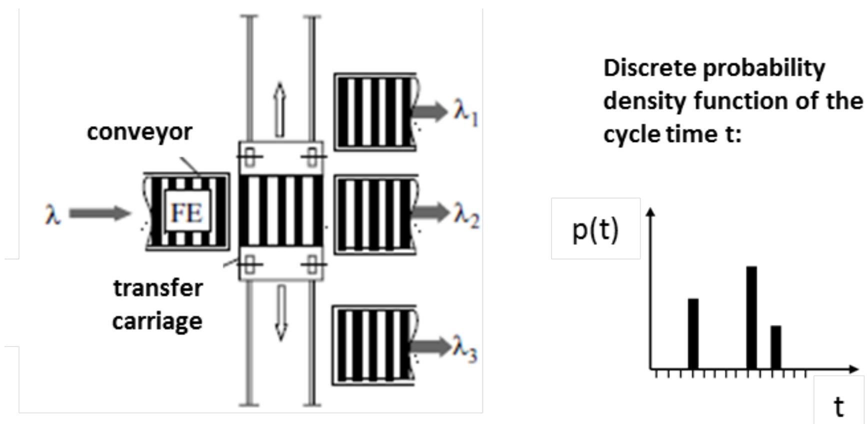

Figure 2: Transfer carriage and discrete-time distribution of the direction-dependent cycle times

the discretization is an advantage, because the level of detail is increased essentially. Discrete time modeling enables not only the derivation of mean values but also the quantiles of performance measure, which are often needed for the design of intra-logistic systems. For example, airport baggage handling systems must be designed in a way that arrival baggage is available within a given time on the baggage reclaims. This has to be ensured e.g. with a probability of more than 95%. The same is valid for lead times of picking orders in a distribution center or the required buffer size in front of a processing station. This is necessary to prevent deadlocks in upstream areas.

Due to the advantages offered by a discrete time approach, numerous models for the basic elements branching, merging [2], single service station [7], and parallel processing stations [4] were developed. In addition, collecting processes and handling of batches were extensively analyzed ([6], [7]). The past research effort has resulted in a wide range of discrete-time calculation methods.

4 Network analysis based on the modular modeling technique

The analysis of an intra-logistic system begins with the computation of results for the single components. The analysis tool provides a so-called heat map mode, which delivers a quick overview of individual characteristics such as utilization, buffer size, or waiting times. This mode converts the numerical results into a color value which visualizes and identifies e.g. the design modules with high utilization or waiting time (Figure 3).

Additional to the probability distributions of the characteristic values for the single design modules, discrete distributions of performance measures for a hierarchical level (e.g. in the warehouse: time between triggering the order of stock removal to provide in the pre-storage area) or for the complete path through a material flow system can be obtained easily. For individual transport relations, the distribution of the lead time can be determined. Moreover, it is also possible to visualize the distributions of the pure transport (depending on distance and velocity), service (depending on the processes) and waiting times (depending on the workload and working methods) of the passed design modules and conveyor lines.

Figure 4 shows the results of an analysis conducted with the analysis tool and visualizes the mean value and quantiles of the lead time for the investigated system. From the figure, it can be seen that e.g. the 95%-quantile of the lead time has a value of 399.9 time units. This means that with a 95% probability an order is completed within a period of up to 399.9 time units.

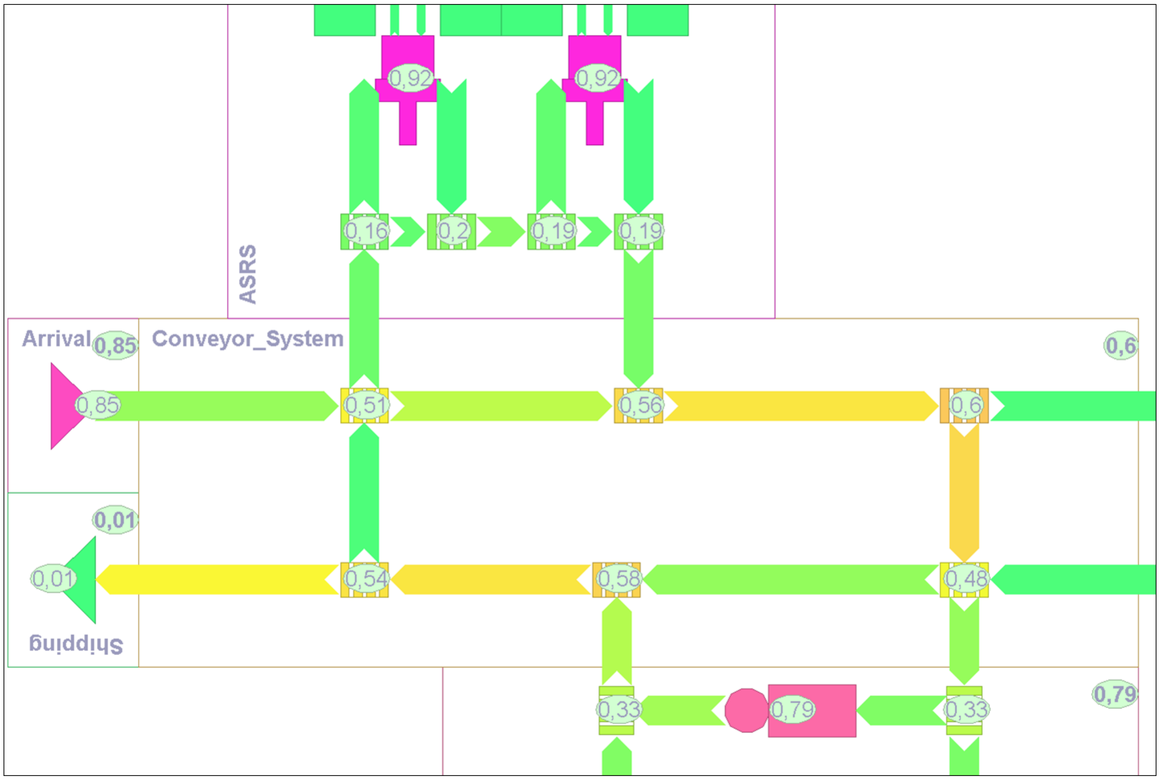

Figure 3: Heat map of the utilization of a pallet transport system: the higher the red component of the individual sections, the higher is the utilization

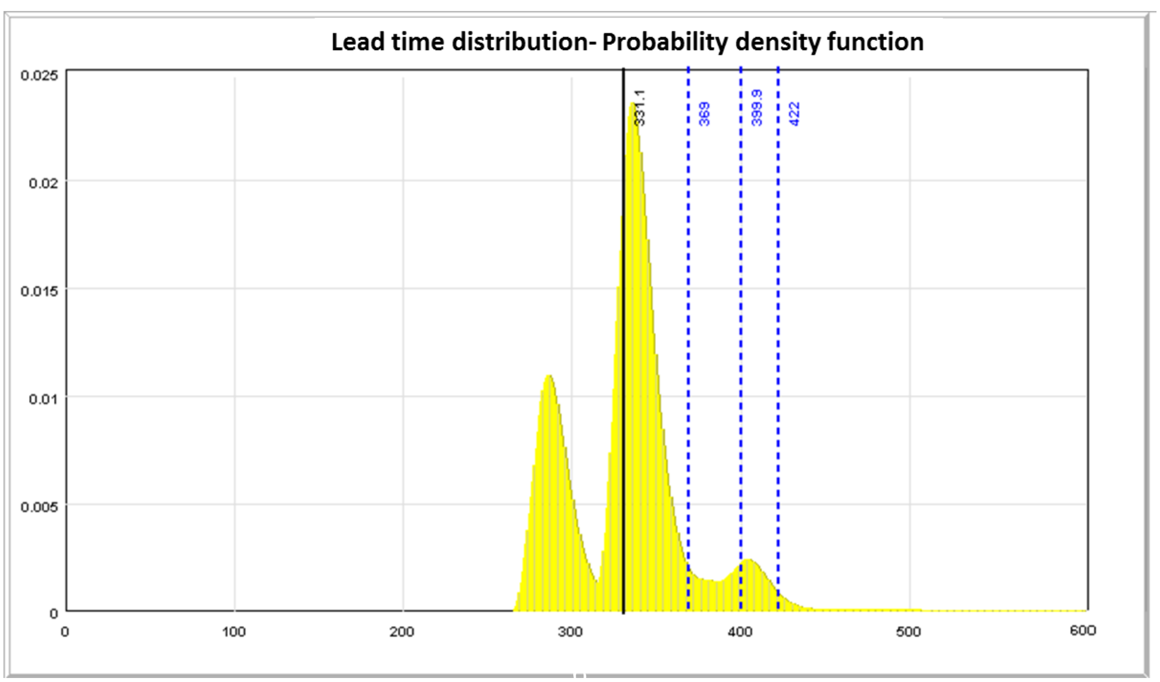

Figure 4: A histogram of the lead time distribution arises as the calculation result. Mean value (solid) and the 90%, 95% and 99%-quantiles (dashed) are marked

5 Identification of missing analytical models in discrete time domain

It is intended to build an extensive component library, in order to provide a wide variety of components for intra-logistic systems. Necessary calculation methods are assigned to the design modules. For a design module representing a transfer carriage, e.g. a kinematic model is required for calculating driving times, another model for calculating limiting throughput, as well as a discrete-time model for the calculation of waiting and departure time distributions. Also different control strategies such as priority rules (absolute priority, gated/time limited service or FIFO) should be taken into consideration to cover a large scope of components of real intra-logistic systems.

In a first step, we identified material handling elements of representative continuous conveying systems. The systems, their technical elements and the material flow elements by which the technical elements can be represented are listed in table 1. An operator station for example can be used to represent an order picking station of a pre-zone of an order picking system. Further systems that were identified are baggage handling systems, post-hubs and tote conveyor system (also known as tray handling systems).

In a second step, we linked these typical material handling elements with existing stochastic analytical models in discrete time domain (see table 2) and identified material flow elements that so far cannot be modeled by the existing models. In the first two columns, the identified material flow elements and possible routing strategies can be found. Columns 3 and 4 show if there are analytical models already existing to model the identified elements and their routing strategies. Which analytical models can be used is shown as well. It can be seen that in the case of the operator station there are already two mathematical models available ($G|G|1$ and $G^x|G|1$) whereas in the case of the continuous merge there are no models in discrete time domain available so far. For one of the missing analytical models, a 4-way-crossing with time windows, we developed a calculation method, which we will briefly present in the next section.

system	technical element	material flow element
pre-zone; order picking from large load carrier	turntable / rotary table	partially continuous divert
		partially continuous merge
		crossing
	corner transfer unit	partially continuous divert
		partially continuous merge
		crossing
	conveyor segment	conveyor
	order picking station	operator station
	ASRS/automated storage and retrieval system	operator station
	transfer carriage	partially continuous divert
		partially continuous merge
		crossing
baggage handling system	loading station	synchronisation station
	divert	continuous divert
	merge	continuous merge
	security check	operator station
	manual encoding	operator station
	EBS (early baggage store)	sink
		source
	ETS (empty tray storage)	sink
		source
	tilter	clone
	tilt tray sorter	continuous merge
		continuous divert
	cross belt sorter	continuous merge
		continuous divert
post hub	conveyor belt	conveyor
	sorter	continuous merge
		continuous divert
	slide	operator station
	telescopic belt conveyor	conveyor
tote conveyor system	circulating vertical conveyor	conveyor
	vertical conveyor	crossing
	pusher	continuous divert
		continuous merge
	pop-up wheel diverter	continuous divert
		continuous merge
	belt deflector	continuous divert
		continuous merge
	diagonal diverter	continuous divert
		continuous merge
	conveyor segment	conveyor
	palletising	collecting station
	depalletising	batch decomposition
	sequencer	stochastic delay

Table 1: Material flow elements for typical material flow systems

material flow element	routing strategy/ queueing principle	existing models					modified models
		batch building capacity rule	G|G|1	Gx|G|1	duplication of stochastic flow	stochastic divert	
batch decomposition	FIFO	x					
operator station	FIFO		x	x			
conveyor	no accumulating capability						x
	accumulating capability						x
	circulating vertical conveyor						x
clone	FIFO				x		
crossing	time window						x
	block handling						x
	round robin						x
	limited right of way						x
collection station	FIFO	x	x				
continuous divert	FIFO					x	
partially continous divert	FIFO		x			x	
continuous merge	time window						x
	block handling						x
	limited right of way						x
	absolute right of way						x
partially continous divert	round robin						x
	limited right of way						x
stochastic delay	FIFO						x
synchronisation station	FIFO						x

Table 2: Analytical models for basic material flow elements

6 Analysis of a 4-way crossing module

In conveying systems, turntables (e.g. rotary table, corner turntable etc.) are often used to divert the conveying units from different sources to different destinations. The number of source directions and destinations varies according to the specific need of the conveying system, in which the turntable is implemented. Often, conveying units are diverted from one single source to one of two possible destinations or are merged from two source directions to a single destination.

This section is devoted to the analysis of a 4-way-crossing with two possible sources and two possible destinations, which is also the most general form of turntables. In the figure below, such a 4-way-crossing is illustrated. Conveying units from directions

1 and 2 are conveyed to the directions 3 and 4. The turnable positions itself in accordance to the direction to be served. The 4-way-crossing is investigated under

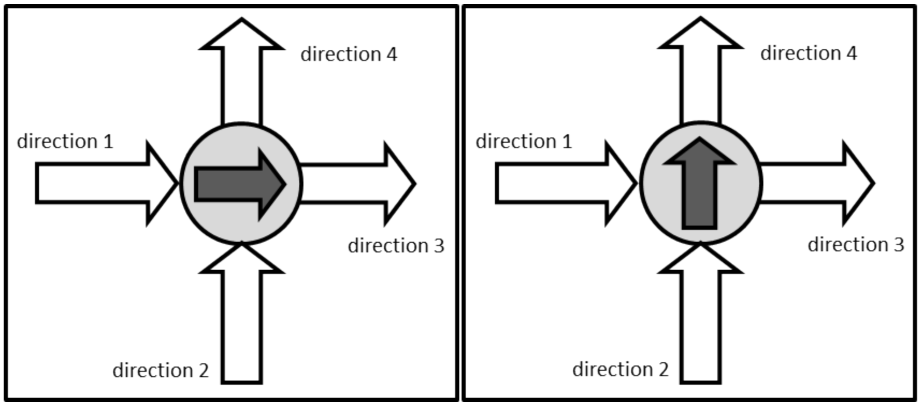

Figure 5: Left: 4-way-cross at position 1; right: 4-way-cross at position 2

the time window strategy (also known as time-limited service discipline). According to this strategy, each source direction is served for a given period of time. After conveying one conveying unit from one direction, the turntable turns back to the given position, as long as the time window for this direction has not passed. For instance, if direction 1 is served, and the next conveying unit has to be diverted to direction 3, the turntable stays at position 1 (see left side of figure 5). We assume that the conveying time from direction 1 to direction 3 is constant and denoted by T_s. If the time window has passed after conveying this unit, the turntable makes a switch movement, which takes S time units, and positions itself to position 2. Otherwise, the turntable waits at position 1 for the next units. Given that the next unit has to be diverted to direction 4, the unit enters the turntable, the turntable switches to position 2 and conveys the unit to direction 4 (see right side of figure 5). Therefore, we assume that the conveying time from a source to a discontinuous direction including the necessary switch is T_u time units. Again, after conveying this unit, the remaining time is checked. If there is still remaining time, an additional switch is made to turn back to position 1. If not, the turntable stays at position 2 and starts serving direction 2. At this point, the time window of direction 2 is initiated. Therefore, the time window strategy adheres to following characteristics:

1. For each source direction, there is a given constant time window.
2. Each incoming unit from a specific direction is conveyed as long as the time window has not passed (in other words, as long as the remaining time is greater than zero).

3. Conveying of a unit is not interrupted if the time window has elapsed during its conveying time. The turntable finishes the conveying of this unit and then positions itself to the other direction. (As a result, the actual time that the turntable serves a direction can be greater than its time window).

In the next section, the derivation of the queue states is explained.

6.1 Derivation of the queue states

We illustrate here the derivation of the queue state at the beginning of the time window of an arbitrary direction. Important variables and parameters used in this analysis are listed below:

A^i	inter-arrival time at direction i
P_{ih}	transition probability from source direction i to direction h
Z^i	time window for direction i
C^i	actual time needed to serve direction i
T_s	conveying time from a source to its continuous direction (e.g. from direction 1 to direction 3 or from direction 2 to direction 4)
T_u	conveying time from a source to its discontinuous direction (e.g. from direction 1 to direction 4 or from direction 2 to direction 3)
S	switch time
B^i	service time of a unit from direction i
X^{+i}	number of conveying units left over at direction i at the end of its time window.
G^i	number of conveying units collected at direction i during the time needed to serve the other direction
X^i	queue state (length), number of conveying units at direction i at the beginning of its time window

Similar to the approach presented in [1], we model the turntable as a polling system, in which one single server serves two queues cyclically, and propose an iterative algorithm to approximate the queue states. The steps of the algorithm are summarized as follows.

(1) Initialization: initialize distributions of X^{+i} and C^i and compute the initial X^i for $i = 1, 2$

(2) Recalculate distributions of X^{+i} and C^i based on the computed distribution of X^i for $i = 1, 2$

(3) Recalculate distribution of X^i for $i = 1, 2$

(4) Repeat steps (2)-(3) until a convergence criterion is fulfilled

Complying with this algorithm, distributions of queue states are updated in each iteration step based on their distributions from the previous step. The algorithm terminates when the given convergence criterion is satisfied. A convergence criterion can be, for instance, the absolute difference between the mean queue state from the actual iteration step and that from the previous iteration step (e.g. we use in our analysis $\mid E(X^i_{n+1}) - E(X^i_n) \mid < 0.0001$ as the convergence criterion).

6.2 Initialization

In this step, we compute the service time distributions for both directions. To do so, we introduce the probability P^i_s that an arbitrary conveying unit from direction i is conveyed to the continuous direction. Similarly, P^i_u is the probability for the discontinuous direction. They are determined as follows:

$$P^i_s := \begin{cases} P_{13} & \text{if } i = 1 \\ P_{24} & \text{if } i = 2 \end{cases} \qquad P^i_u := \begin{cases} P_{14} & \text{if } i = 1 \\ P_{23} & \text{if } i = 2 \end{cases}$$

Based on the given probabilities, the service time distribution for the conveying units can be computed as follows.

$$\begin{aligned} P(B^i = T_s) &= P^i_s \\ P(B^i = T_u + S) &= P^i_u \end{aligned}$$

The given service time distribution applies for all conveying units that are handled before the last conveying unit in a time window. For such conveying units, the remaining time is always positive. Therefore, the turntable has to position itself to the original position after the unit is conveyed. So, an extra switch time can arise, which is considered in our analysis as a part of the service time. As a result, the conveying time for the continuous direction always equals T_s. In this case, the turntable stays at the original position. For the discontinuous direction, the conveying time is the sum of T_u and switch time S, as the turntable must go back to the original position after conveying the unit. However, for the last conveying unit that is handled in a time window, the service time distribution may differ. If the time window has elapsed after conveying the last unit, the turntable has to position itself to the other source. In this case, a switch has to be made after conveying to the continuous direction, yielding a total service time of $(T_s + S)$ time units, whereas the service time for the discontinuous direction is T_u.

Subsequently, the distributions of X^{+i} (the number of unit that are left over in the queue after the time window for the source direction has elapsed) and C^i (total time needed to serve the given source direction) are initiated as follows:

$$P(X^{+i} = 0) = 1 \qquad \forall i = 1, 2 \tag{1}$$

$$P(C^i = Z^i) = 1 \qquad \forall i = 1,2 \tag{2}$$

In order to determine G^i, we firstly introduce an operator, which computes the distribution of the number of arrivals in a given time interval. With this operator, the conditional probability ($P(G^i = l \mid C^{i'} = m)$), that l arrivals are observed in a time period of m time units, is computed. For different values that the actual time needed to serve the other direction ($C^{i'}$ where $i' \neq i$), conditional probabilities of G^i are computed. With the law of total probability, we attain:

$$P(G^i = l) = \sum_{m=Z^{i'}}^{c_{max}^{i'}} P(G^i = l \mid C^{i'} = m) \cdot P(C^{i'} = m) \qquad \forall i' \neq i \tag{3}$$

where $c_{max}^{i'}$ is the maximum value of $C^{i'}$.

The queue state is the sum of X^{+i} (number of units left over in the queue after the time window of the source direction has elapsed) and G^i. Hence, we obtain the initial queue states (X^i) as the convolution of X^{+i} and G^i:

$$X^i = X^{+i} \otimes G^i \qquad \forall i = 1,2 \tag{4}$$

6.3 Derivation of X^{+i} and C^i

In order to derive the distributions of X^{+i} and C^i, we differentiate between three cases:

> Case 1: All units in the queue at the beginning of the time window of a given direction (X^i) are conveyed. Additionally, a part or all of the new arrivals during the time window is conveyed.
>
> Case 2: Just a part or all of X^i is conveyed during the time window. No new arrival is conveyed either because there was no remaining time left or there was no new arrival during the time window.
>
> Case 3: There were no units in the queue at the beginning of the time window ($X^i = 0$) and no new arrival occurred during the time window.

For case 1, we introduce the variables arrival time T_j^i and completion time K_j^i for each arbitrary j-th arrival within the time window. The arrival time T_j^i is the time between the start of a time window and the arrival of the j-th conveying unit from the source direction i and K_j^i is the time interval between the start of the time window and the completion of service of the j-th unit (see figure 6).

In order to compute the arrival and completion times for new arrivals, we firstly compute the service completion time for the units already in the queue (X^i). For this purpose, we use the service distribution introduced above, which applies to conveying

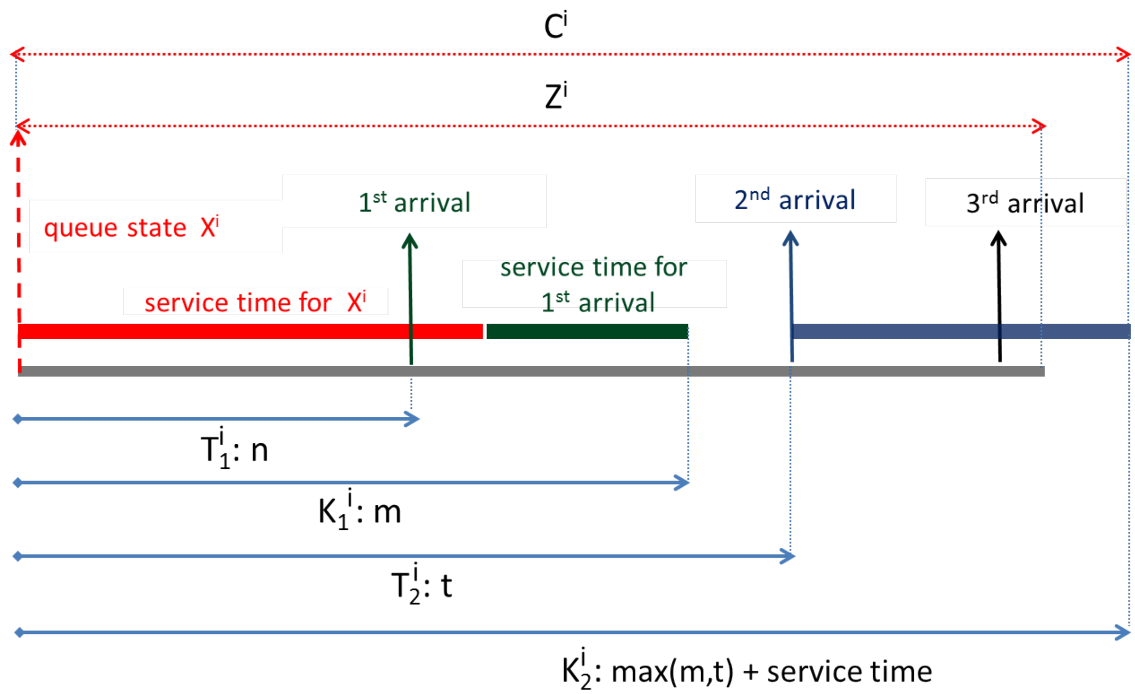

Figure 6: Illustration of case 1

unit handled before the last unit served in a time window. The reason is that none of the units from queue existing at the beginning of the time window can be the last unit under case 1, in which at least one new arrival is conveyed. Based on the service completion time distribution for X^i, the joint probability of service completion time and arrival time is derived for the first arrival. The service of the first arrival starts either at the time point of service completion for X^i, if the arrival has already occurred at this point of time. If not, the turnable waits idle till the first arrival and service starts immediately. The service completion time for the first arrival is the sum of its service start time and the service time for this unit. Dependent on the completion and arrival time distributions for the first arrival, we can now compute these of the second arrival. For further arrivals, we proceed similarly and employ the completion and arrival time distributions for the previous arrival.

Figure 6 illustrates the derivation of the joint probability of completion and arrival times on the example of the second arrival. The start of the service for this unit is either its own arrival time (t) or the service completion time of the previous arrival (m), whichever is greater. Consequently, the service completion time for the second arrival is the sum of service start time ($\max(m, s)$) and its own service time. The service time of a unit depends on two criteria: the destination (continuous/discontinuous) of the unit and the remaining time after conveying the unit. We display here the computation of the joint probability of completion and arrival time for the j-th unit that is conveyed to the continuous direction:

$$
\begin{aligned}
&P(K_j^i = (\max(m,t) + T_s + x \cdot S) \wedge T_j^i = t) \\
&= \sum_{n=1}^{t-1} \sum_{m=n+1}^{Z^i-1} P(K_{j-1}^i = m \wedge T_{j-1}^i = n) \cdot P(A^i = \text{t-n}) \cdot P_s^i
\end{aligned}
$$

where

$$
x = \begin{cases} 1 & if \quad (\max(m,t) + T_s) \geq Z^i \\ 0 & else \end{cases} \tag{5}
$$

In this formula, it is assumed that the previous unit had the arrival time n and the completion time m. If the next inter-arrival time has a value $(t-n)$, then the current unit has the arrival time t and the completion time is the sum of $(\max(m,t))$ and its service time. If the unit is conveyed to the continuous direction, its conveying time is T_s and the probability for this is P_s^i. In the formula, we introduced the variable "x". This variable accounts for the additional switch time needed in the case that the time window passed after the unit was conveyed. In this case, the turntable has to position itself to the reverse source direction after conveying the unit. Therefore, an additional switch time is needed, which is considered in the equation as a part of the service time for the conveyed unit. Finally, we consider all the possible values of m and n. The equation for a unit to be conveyed to the discontinuous direction is derived analogously.

After computing the joint probabilities, we calculate the number of units left over in the queue (X^{+i}) under case 1 by considering two possibilities, in which $(X^{+i} > 0)$. Under the first possibility, an arbitrary j-th arrival happens in t time units and its service is completed in z time units after the start of the time window, where $z \geq Z^i$. Consequently, all the new arrivals in time interval $(z-t)$ will be left over. For instance, the third arrival in figure 6 will be left over in the queue. For this possibility, we compute the number of arrivals within the time interval $(z-t)$.

It is also possible that the service of j-th arrival is completed before the end of the time window $(z < Z^i)$ and no additional arrival occurs till the end of the time window. At the end of the time window, the turntable is still at the original position and has to switch to the other position, which takes an additional S time units. If there are new arrivals within S time units, these arrivals cannot be conveyed till the next time window for this direction and have to wait in the queue. Considering these two possibilities, the distribution of X^{+i} is attained.

Thereafter, we compute the actual time needed for direction i (C^i) under the first case. We compute the probability that C^i has a value greater than Z^i. C^i is greater than Z^i if the service completion time of the last unit handled exceeds Z^i. Another possibility is that the completion time of the last unit handled was less than Z^i and no additional arrival occurred after this unit. As a result, the turntable positions itself to the reverse source direction after the time window is over, yielding $C^i = Z^i + S$. Using joint probabilities for completion and arrival times, we can compute the distribution of C^i under the first case. Subsequently, X^{+i} and G^i are calculated under cases 2 and 3. Afterwards, their distributions can be determined as follows:

for $n \geq 1$

$$P(X^{+i} = n) = P^1(X^{+i} = n) + P^2(X^{+i} = n) + P^3(X^{+i} = n) \tag{6}$$

where $P^1(X^{+i} = n)$, $P^2(X^{+i} = n)$, and $P^3(X^{+i} = n)$ are the probabilities that X^{+i} is n units under cases 1, 2, and 3, respectively. Subsequently, it yields:

$$P(X^{+i} = 0) = 1 - \sum_{n=1}^{\infty} P(X^{+i} = n) \tag{7}$$

We proceed similarly for C^i:

for $p \geq 1$

$$P(C^i = Z^i + p) = P^1(C^i = Z^i + p) + P^2(C^i = Z^i + p) + P^3(C^i = Z^i + p) \tag{8}$$

$$P(C^i = Z^i) = 1 - \sum_{p=1}^{\infty} P(C^i = Z^i + p) \tag{9}$$

6.4 Derivation of X^i

Eventually, we compute the queue state distribution in the way, as we did in the initialization. We again compute the distribution of G^i based on the new distributions of $C^{i'}$ (see equation 3) and convolute it with X^{+i} (see equation 4). In this way, we attain the queue state distribution and use it to check the convergence criterion.

6.5 Numerical results

As we have presented an approximate method for the queue states, we discuss here the quality of the algorithm introduced. For a range of parameters, which are tested up to now, the algorithm yields very accurate results for the higher moments and the quantiles and generally the computed distributions follow the actual distributions accurately. An example is provided in appendix, in which we compare the analytical results for queue states with the simulation results (see tables 3 to 5 for inputs). Results for the mean, 95%- and 99%- quantiles as well as relative deviations are displayed (see table 6). Analytical model delivers the right quantiles. However, the mean values show relative deviations of around 4%. Moreover, distributions of the queue states are visualized in appendix, which follow each other accurately (see figure 7). Although results are promising, more experiments are needed to quantify the quality of the method. After this step, inter-departure and waiting time distributions can be modeled, which depend on the queue state. Eventually, the model for the 4-way-cross under the time window strategy can be integrated as a design module to the analysis tool.

7 Conclusion

The aim of the research project is to develop efficient methods, in order to support the decision making process in the early planing phase of complex intra-logistic networks. For this purpose, typical technical elements, that are installed in intra-logistic networks, are identified. For some of these elements, already existing analytical models could be implemented. However, new models have to be developed for the majority of technical elements. Moreover, the quality of the analytical approach has to be studied. In particular, the effects of implemented control strategies and possible blockages in the network must be addressed, which cannot be captured exactly by analytical approaches.The same applies for the analytical treatment of transient states, which often occur in highly dynamic systems (e.g. if the external load changes often before the steady state can be reached) and the possible correlations (e.g. autocorrelated departure times). The deviation caused by such points must be reduced e.g. by introduction of correction factors.

In addition to the accuracy of the approach, another point to address is the performance of the numerical methods. It is well-known that the computing time may increase drastically for large networks through the repeated application of statistical operations (e.g. convolution) and increasing vector sizes. Therefore, strategies must be developed to manage the computational complexity.

Appendix: numerical results

Inter-arrival Time	Direction 1	Direction2
Value	Probability	Probability
6	0.5	0
7	0	0.5
8	0.5	0.5

Table 3: Analysis of a 4-way crossing module; inter-arrival time distributions

Transition probabilities		
Direction	3	4
1	0.6	0.4
2	0.6	0.4

Table 4: Transition probabilities

Times for the turnable		Time windows	
T_s	1	Z^1	10
T_u	2	Z^2	10
S	1		

Table 5: Left side: operational times for the turnable; right: durations of time windows

	Queue 1			Queue 2		
	analy	sim	Δ rel.	analy	sim	Δ rel.
mean	1.81	1.88	0.04	1.74	1.80	0.04
95%-quantile	3	3	0.00	3	3	0.00
99%-quantile	3	3	0.00	3	3	0.00

Table 6: Results summary for the queue states; relative deviations of the mean, 95%- and 99%-quantiles

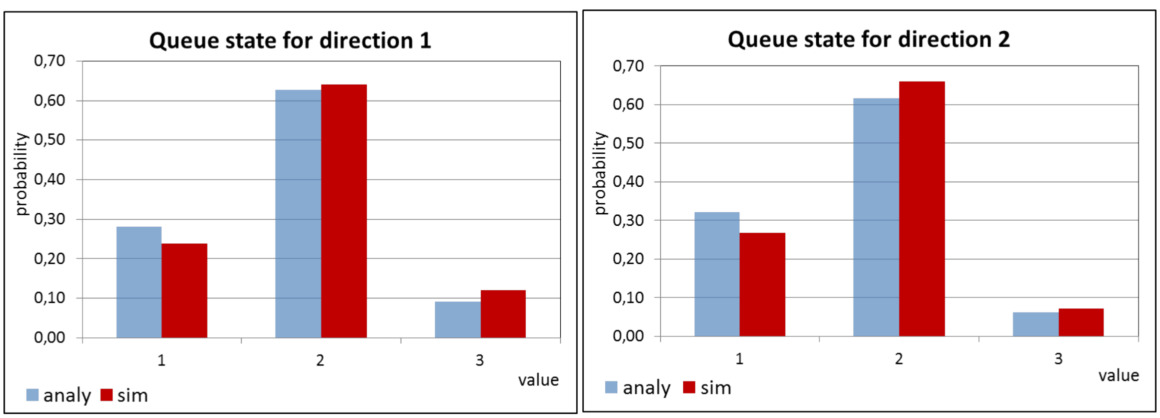

Figure 7: Analysis of queue states; comparison of analytical results with simulation results

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