

# The order picking problem in fishbone aisle warehouses

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## Abstract

A recent trend in the layout design of unit load warehouses is the application of layouts without conventional parallel pick aisles and straight middle aisles. Two examples for such designs are flying-V and fishbone designs for single and dual command operations. In this study, we consider the same layout types under the case of multiple-item pick lists and show that, for both layout types, the routing problem can be solved in polynomial time. We also propose simple heuristics for this problem inspired by those put forward for parallel-aisle warehouses. Our computational results reveal that under certain cases, fishbone design can perform as high as 30% worse than an equivalent parallel-aisle layout, and a modification of the aisle-by-aisle heuristic produces good results compared to other heuristics.

## 1 Introduction

An important aspect regarding efficient and effective management of supply chains is the management of warehouses, which, according to Bartholdi and Hackman [1], serve two main purposes. The first is to better match supply with demand, as warehouses can allow quick response to surges in demand such as seasonality and changes in supply such as quantity discounts. The second purpose is to reduce transportation costs by consolidating product and eliminating unnecessary travel.

A typical warehouse generally performs four main functions: receiving, put-away to storage, picking

and shipping. Tompkins et al. [14] have broken down the warehouse operation costs in the United States and have found out that more than half of these costs can be attributed to order picking activities. A further analysis of the time spent for picking reveals that around half of the time spent during order picking is due to travel time of the picker. This emphasizes the importance of efficient routing of pickers on the responsiveness of a warehouse. Motivated by this, the order picking problem (OPP) has been defined to be one of finding the shortest route that starts at a *pickup and deposit* (P&D) point, also called a *depot*, picks all the items in the picking list, and returns to the P&D point.

Figure 1 shows a typical parallel-aisle warehouse, which will be interchangeably referred to in this paper as a *traditional* warehouse. In a traditional warehouse, the items are located on parallel *pick aisles*, lying perpendicular to the *front* and *back cross aisles*, and possibly *middle aisles*, which make traveling between the pick aisles easier. In the existence of middle aisles, the warehouse is divided into *blocks*, and the pick aisles are further divided into *subaisles*. We will refer to the OPP with  $r-1$  blocks as *r-OPP*.

A recent trend in design of warehouse layouts is the consideration of different designs for the pick aisles and middle aisles. Gue and Meller [5] analyze the underlying assumptions in warehouse layout design, and observe that there are two “unspoken rules” in laying out a warehouse, which state that the pick aisles should be straight and perpendicular to the cross aisles, and that any middle aisles, if they exist, should be straight. Relaxation of the latter assumption for a unit-load warehouse applying single-command operations leads to a new warehouse design, called the *flying-V* design, called so because of the fact that the middle aisle extends from the P&D point to the left- and rightmost pick aisles in a V-shape. Next, they relax the first assumption and allow vertical and horizontal orientations of pick aisles to come up with the *fishbone layout*, an example for which is given in Figure 2. In this case, a middle aisle is extended from the depot, and the pick aisles, which can be perpendicular or parallel with respect to the cross aisles, originate from the middle aisle to the cross aisles.

Both flying-V and fishbone layout designs are advantageous in single-command operations, as they reduce the travel distance between the P&D point and the items, compared to an equivalent traditional design with a middle aisle. Pohl et al. [9] consider dual-command operations in a unit-load warehouse, and observe that fishbone design still outperforms traditional layouts on average. However, in most cases, these types of layouts increase the travel between a pair of items as the number of items in the pick list increases. In

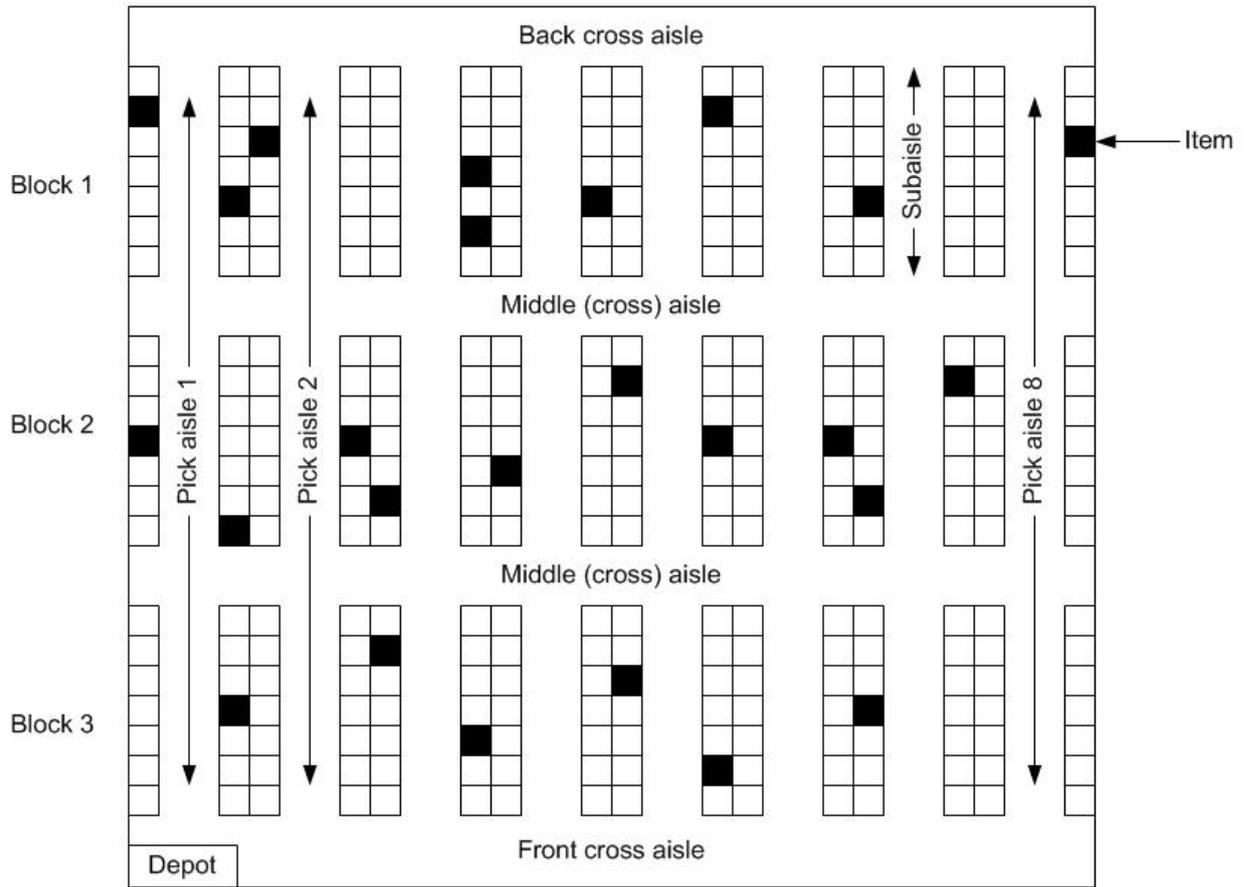


Figure 1: A traditional warehouse with 8 pick aisles, 2 middle aisles, and 25 pick items

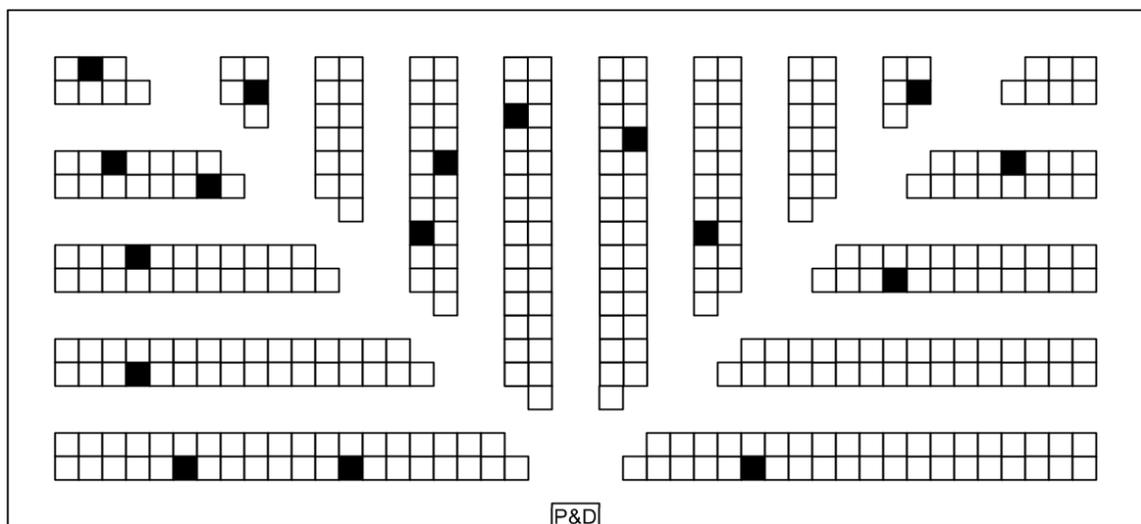


Figure 2: A fishbone warehouse with 11 pick aisles, and 16 pick items

this paper, we consider the case of multiple-item pick lists, and analyze various routing schemes in fishbone layouts. In particular, we aim to answer the following questions:

1. What is the size of the pick list under which a traditional layout starts outperforming a fishbone layout?
2. How can the picking route be optimized in a fishbone layout in the case of multiple-item pick lists?
3. Can we find simple near-optimal routes in a fishbone layout with multiple-item pick lists?
4. What is the effect of varying depth/width ratio of the warehouse in terms of relative performances of the two layout schemes?

The outline of the rest of this paper is as follows: In the next section, we review the literature on the OPP in traditional warehouses as well as new trends in warehouse layout design. In Section 3, we present an algorithm to solve the OPP in fishbone warehouses to optimality. In Section 4, we propose various heuristic procedures inspired from those put forward in the literature for the traditional layout. Section 5 describes and gives results on the computational experiments conducted in order to compare the performances of fishbone and traditional layouts under optimal routing as well as performances of the heuristics. The last section concludes the paper and gives potential future research areas.

## **2 Literature Review**

The work in this study mainly relates to two streams of the literature on warehouse management. The first of these is the literature on routing order pickers in a warehouse, whereas the second stream of works analyzes various layout design schemes.

The seminal work on the optimal routing of order pickers in a warehouse is due to Ratliff and Rosenthal [11], who consider a single-block parallel-aisle warehouse (2-OPP). They show that this problem, which is a special case of the well-known NP-hard Traveling Salesman Problem (TSP), is polynomially solvable. To show this, they present a dynamic programming-based algorithm that runs in linear time in terms of the number of aisles. The main intuition behind the optimal solution procedure is based on the fact that there are a limited number of move types within an aisle and between neighboring aisles. Sequences of these moves

lead to what Ratliff and Rosenthal call the concept of *equivalence classes*, which are triplets formed by the possible degrees of the corner nodes of the pick aisles and the number of connected subtours in the current partial solution. For the case of 2-OPP, they find out that there are 7 possible equivalence classes that need to be considered. The basic idea is to use an approach that starts from the left-most pick aisle, enumerate all possible equivalence classes for each pick aisle, coming up with the best equivalence class solutions for the right-most pick aisle, and determine the optimal route by backward recursion from these.

In theory, the algorithm by Ratliff and Rosenthal [11] can be extended to handle the cases with any number of middle aisles. Roodbergen and De Koster [12] have made such an extension for the case of a single middle aisle (3-OPP). The definition of an equivalence class is extended to include the degrees of the corner nodes on the middle aisle as well. For this problem, the number of possible equivalence classes increases to 25. The solution procedure is similar to that by Ratliff and Rosenthal, except that the procedure is first applied to the lower block, then it proceeds with the upper one. The optimal solution can be found by backtracking from the best equivalence class solution for the right-most pick aisle of the upper block. For the cases with more than one middle aisle, the number of possible equivalence classes increases exponentially, hence it becomes of little practical use to extend the algorithm for those.

Due to the enumerative nature of the above approaches and the need to come up with more simple routes for the pickers to follow, various heuristic procedures have been proposed for both 2-OPP and  $k$ -OPP with  $k \geq 3$ . Hall [7] describes a set of simple heuristic procedures for 2-OPP. Among these, *S-shape* heuristic starts with the left-most pick aisle and traverses in order each nonempty pick aisle completely except possibly the right-most one, resulting in a serpentine path. The *largest gap* heuristic calculates, within each pick aisle, the maximum of the largest gap between any two items (1), the distance between the front corner and the item nearest to it (2), and the distance between the back corner and the item nearest to it (3). The picker starts traversing the left-most pick aisle completely, enters from the back cross aisle the pick aisles for which (1) and (2) are maximum, picking the items until the largest gap, and returning to the back cross aisle. After traversing the right-most nonempty pick aisle completely, the same procedure is applied from the front cross aisle for the pick aisles for which (1) and (3) are maximum.

For warehouses with middle aisles, Vaughan and Petersen [15] propose a dynamic programming-based *aisle-by-aisle* heuristic, which takes pick aisles as stages. A simplifying assumption in this procedure is

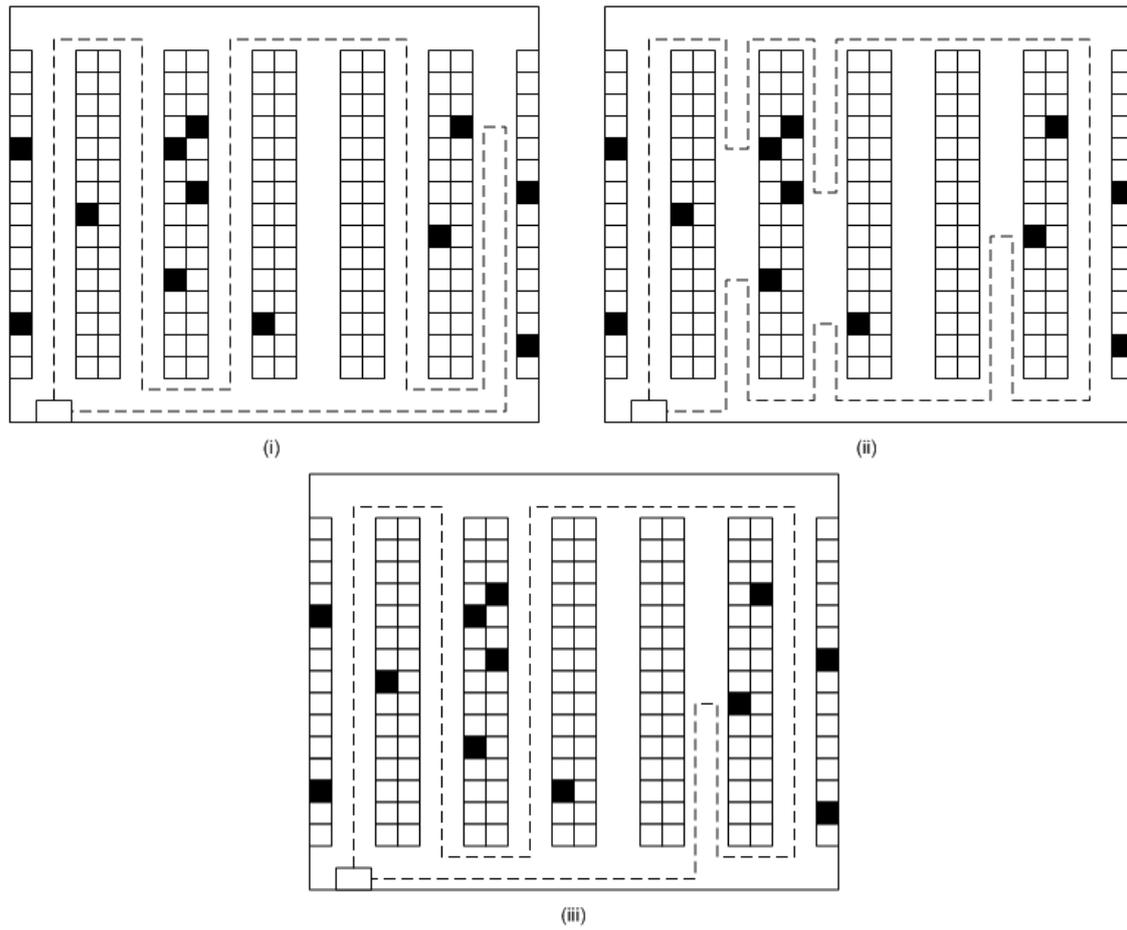


Figure 3: (i) S-shape heuristic solution, (ii) Largest gap heuristic solution, (iii) Combined heuristic (and optimal) solution for an example 2-OPP instance

that a picker can enter and exit a pick aisle only once. The main decision is which middle (or back/front cross) aisle to enter and exit the pick aisle. It should be noted here that for 2-OPP, there are two alternatives for each entrance and exit, namely the front and back cross aisles. Roodbergen and De Koster [13] extend the heuristics described in Hall [7] to the case with middle aisles. They also propose a *combined* heuristic, which is an extension of aisle-by-aisle in that it also allows for entering and exiting a pick aisle more than once, but the subaisles can still be entered exactly once. Based on this fact, the algorithms come up with identical solutions when no middle aisles are present.

Figure 3 illustrates an example 2-OPP instance, for which the S-shape, largest gap and combined heuristic solutions are given. The latter heuristic happens to find the optimal solution for this specific instance.

To the best of our knowledge, the first mention of relaxing the two “unspoken rules” in warehouse design is due to Berry [2], who observes that diagonal pick aisles with varying storage depth can be advantageous over a rectangular block design when dedicated storage is applied, or when the cost of accessibility is lower compared to cost of space. Another relaxation is by White [16], who considers radial aisles projecting away from the P&D point, and gives expected travel distances given a particular layout design with radial aisles.

In their study that relaxes the two unspoken rules, Gue and Meller [5] take a more prescriptive approach. For both relaxations, their model takes as input the number of pick aisles, lane depth, pick aisle width and length as well as the width of the middle aisle, and gives as output the middle aisle should intersect the pick aisles. The model that relaxes the straight middle aisle assumption comes up with a V-shaped middle aisle with an increasing intersection angle with the pick aisles as one moves away from the P&D point. This gives the name “flying-V” design to the layout. The model relaxing the parallel pick-aisle assumption is confined to cases where only vertical and horizontal orientations of pick aisles are possible, and the middle aisle extends in two diagonals from the P&D point. This yields a “fishbone” type layout, which promises reduction in travel distances by 20% in some cases, compared to rectangular block design. Asymptotic analysis reveals that this reduction gives travel time close to the lower bound.

Focusing on fishbone layouts, Pohl et al. [9] extend the work by Gue and Meller to the case of dual command operations. As in the single command case, they find the optimal angle that the middle aisle should intersect the pick aisles in order to minimize expected travel time. In comparison to a traditional design with the same amount of storage space, the reduction in travel time is around 10-15%. A further extension that allows both middle aisles and pick aisles to take any angle is given by Öztürkoğlu et al. [8]. Here, given the number of middle aisles, pick aisles and warehouse dimensions, the objective is to find the optimal angles of the pick aisles and middle aisles to minimize expected travel time for single command operations. For the cases with single, two and three middle aisles, closed form optimality equations are found and the optimal layouts are named as Chevron, leaf, and butterfly, respectively. Gue et al. [6] investigate the performance of flying-V layouts and inverted-V layouts with multiple P&D points and single command operations. They observe that even though the performance diminishes over the single P&D setting, the designs still outperform rectangular block layout. Pohl et al. [9] analyze the performances of flying-V and fishbone designs in the case of turnover-based storage, which shows that a warehouse that is designed for random storage (or a specific demand pattern) will also be a good design if the storage policy (or demand

pattern) changes.

To our best knowledge, there exists only one study that discusses the performance of fishbone layouts under multiple-item pick lists, which is due to Djukić and Opetuk [4]. While the structure of the analysis is similar to the one applied in this paper, they do not consider the optimal routes as a basis of comparison, but rather make the analysis based on the results of S-shape heuristic modified for fishbone layouts. Furthermore, their results do not take into account the effects of dimensions of the warehouse, focusing on a specific choice of warehouse width and length. In this paper, we fill this gap by presenting a polynomial time algorithm that optimally solves the picker routing problem, and also present alternative heuristic methods for the same problem. Our computational comparisons depend on the relative performances with respect to the optimal routes rather than heuristic results.

### 3 An exact algorithm for the OPP in warehouses with fishbone layout

In this section, we provide a polynomial time algorithm that solves the OPP on warehouses with fishbone layout. The main idea behind this algorithm is to transform the fishbone layout graph into an equivalent 3-OPP graph. Due to the polynomial solvability of this problem by Roodbergen and De Koster [12], showing the validity and tractability of the transformation will be sufficient.

The graph representation of any OPP instance on a rectangular warehouse is as follows: Each item  $k$  is represented by a vertex  $v_k$  and  $v_0$  represents the P&D point. Intersections of the back cross aisle with pick aisle  $i$  are denoted by vertices  $a_i$ , whereas corners of the front cross aisle with each pick aisle  $i$  are represented by vertices  $b_i$ . If any middle aisles exist, corners of pick aisle  $i$  with middle aisle  $j$  (with  $j = 1$  being the closest to the back cross aisle and so on) are represented by vertex  $m_{ij}$ . The edges represent direct accessibility between items, from a corner to an item, and vice versa. To provide an example, Figure 4 gives the graph representation of the OPP instance in Figure 1.

We use a similar graph representation for the fishbone layout, for which, without loss of generality, we assume a symmetric structure with the P&D point being in the middle of the front cross aisle. As an example, the graph representation of the instance in Figure 2 is given in Figure 5. Once again, each item  $k$  and the P&D point are represented by vertices  $v_k$  and  $v_0$  respectively. Assume the pick aisles are labeled in

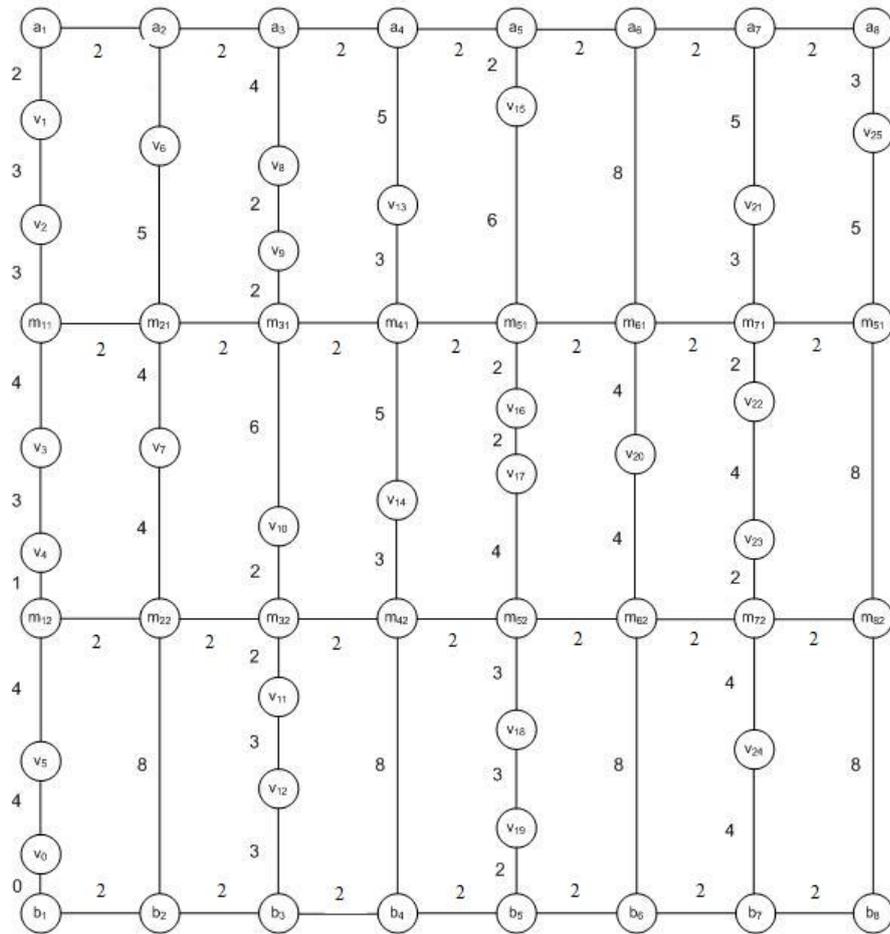


Figure 4: Graph representation for the warehouse in Figure 1

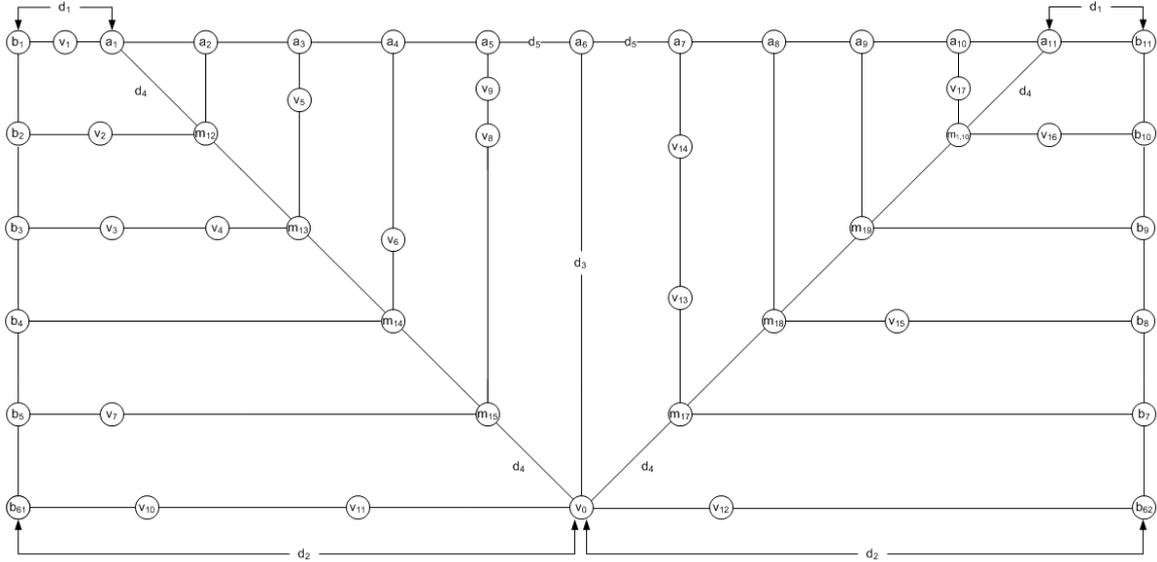


Figure 5: Graph representation of the fishbone layout in Figure 2

the following way for the left half: Pick aisle 1 is the back-most pick aisle. The  $i^{th}$  pick aisle consists of the  $i - 1^{st}$  vertical subaisle from the left and  $i^{th}$  horizontal subaisle from the back. Assuming  $n$  aisles have been labeled in the left half, the vertical aisle in the middle, together with the two front-most subaisles, forms the  $n + 1^{st}$  aisle. For the right-half, the  $i^{th}$  pick aisle consists of the  $i - 1^{st}$  vertical subaisle from the left and  $i - n^{th}$  horizontal subaisle from the front, except the  $2n + 1^{st}$  aisle, which is the top-most subaisle on the right half.

Based on the aforementioned labeling of pick aisles, we label the intersection of each pick aisle  $i$  and the middle aisle by  $m_{1i}$ . The back-most corner of each pick aisle  $i$  is represented by a node  $a_i$  and the front-most corners by  $b_i$ , except for the  $n + 1^{st}$  pick aisle, for which the front corner on the left half is labeled  $b_{n+1,1}$ , and the front corner on the right half is labeled  $b_{n+1,2}$ . The length of the first and  $2n + 1^{st}$  pick aisles are both given as  $d_1$ , the subaisles on the front cross aisle are of length  $d_2$ , the depth of the warehouse is taken as  $d_3$ , the distance between two adjacent  $m_{1i}$  nodes is  $d_4$ , and two adjacent  $a_i$  nodes are at  $d_5$  distance from each other. We will ignore the rest of the distances on the graph as they will not be of importance during the graph transformation. Due to the symmetry of the warehouse graph, we will focus on only the left half of the transformation throughout the rest of this section.

The transformation starts by unfolding the first and  $n + 1^{st}$  pick aisles to eliminate the possibility of pick

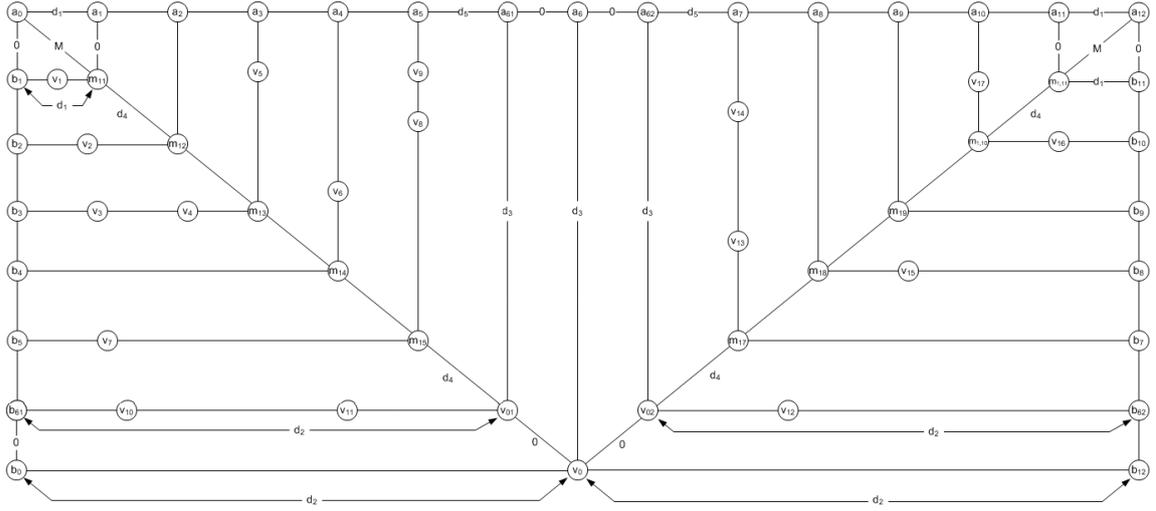


Figure 6: Graph representation of the fishbone layout in Figure 2 after the creation of extra cross aisles

items from being on the front and back cross aisles. To unfold the first pick aisle, we first copy nodes  $b_1$  and  $a_1$ , and name these nodes  $a_0$  and  $m_{11}$  respectively. We then create two edges of zero length, one between  $a_0$  and  $b_1$ , and one between  $a_1$  and  $m_{11}$ . Following this, we join  $a_0$  and  $a_1$  by creating an edge of length  $d_1$ , and join  $a_0$  and  $m_{11}$  by creating an edge of sufficiently large length so that the picker never uses it.

To unfold the  $n + 1^{st}$  pick aisle, we first make a copy of  $a_{n+1}$  at zero distance on the edge joining it to  $a_n$ , and name this new node  $a_{n+1,1}$ . We then create a copy of  $v_0$  at zero distance on the edge connecting it to  $m_{1n}$ , and name this node  $v_{01}$ . We connect the  $n + 1^{st}$  pick aisle to it instead of  $v_0$ , and also connect it to  $a_{n+1,1}$  using an edge of length  $d_3$ . Lastly, we create a copy of  $b_{n+1,1}$  and name it  $b_0$ . We create an edge of length zero between  $b_0$  and  $b_{n+1,1}$ , and an edge of length  $d_2$  between  $b_0$  and  $v_0$ .

The resulting graph after the first step for the example in Figure 2 is given in Figure 6.

To separate the P&D node from the middle aisle, we create another copy of it, labeled  $v_{03}$ , at zero distance from it on the edge joining it to  $a_{n+1}$ , and connect the edge coming from  $v_{01}$  to  $v_{03}$  instead of  $v_0$ . Following this, all  $b_i$  nodes except  $b_0$  are moved onto the artificial front cross aisle created in the first step. It should be noted here that keeping  $b_0$  on the corner does not change the pairwise distance between any pair of nodes. The distance between  $v_0$  and  $b_{n+1,1}$  is  $d_2$ , and all the remaining distances between pairs of  $b_i$  nodes are kept the same. Figure 7 gives the resulting graph after these modifications are made to the example in Figure 2.

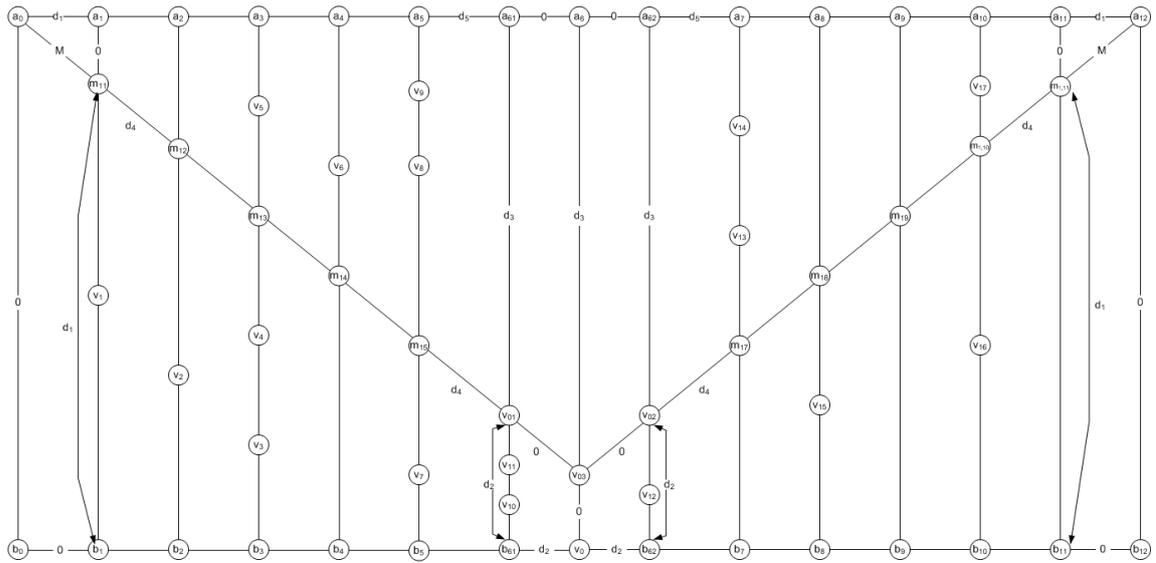


Figure 7: Graph representation of the fishbone layout in Figure 2 after the duplication of the P&D node and moving of the  $b_i$  nodes to the front cross aisle

For the last step, an artificial middle aisle node,  $m_{10}$  created (as a copy of  $a_0$ ) on the left-most pick aisle at zero distance from  $a_0$  and  $b_0$ , and the edge connecting  $m_{11}$  to  $a_0$  is connected to  $m_{10}$  instead of  $a_0$ . The transformed graph for the example in Figure 2 is given in Figure 8.

It is obvious from the above procedure that the transformation can be completed in polynomial time in terms of the number of pick aisles, and the complexity of this procedure does not depend on the number of pick items. The validity of the transformation can also be easily verified from the fact that the pairwise distances between the nodes in the original fishbone graph and the corresponding nodes on the resulting 3-OPP graph are identical. One other aspect of this transformation to be mentioned is the fact that the extended graph conserves planarity of the original one.

What remains to be shown is that the transformation is valid for any type of fishbone warehouse. There are two more possible differences for the layout of a fishbone warehouse compared to the one discussed in the example, the first of which occurs when the ends of the pick aisles do not meet on the middle aisle, or when horizontal or vertical parts of the pick aisles are missing, as in both of the warehouse graphs in Figure 9. This can be overcome by adding infinite-length horizontal or vertical aisles to those ends depending on which part of the aisle is missing. Secondly, the middle aisle may end at the left and right end of the warehouse, rather than at the back cross aisle (as in the first warehouse in Figure 9). In this case, the

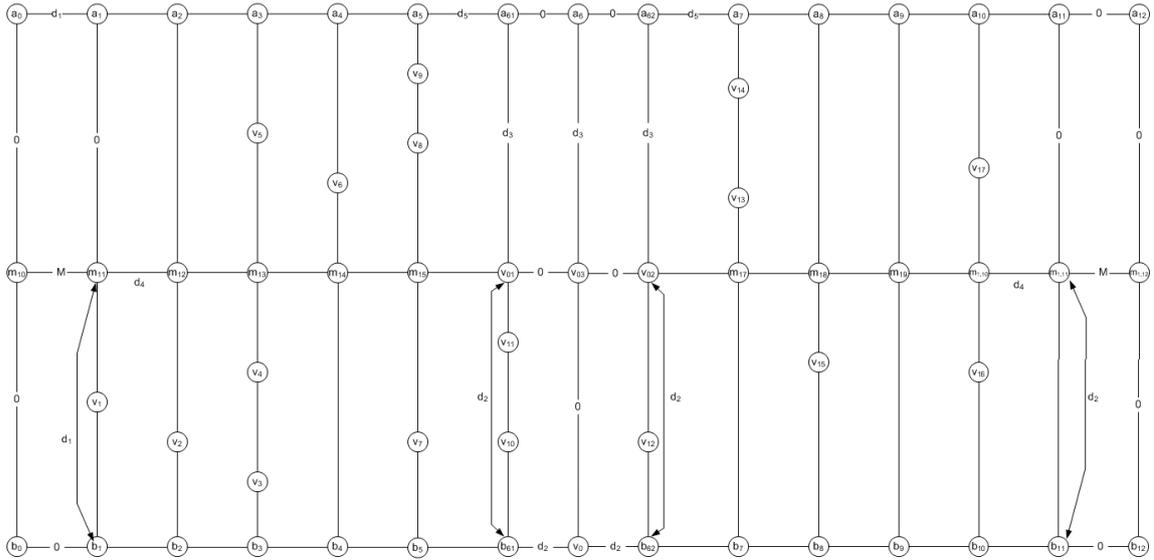


Figure 8: Resulting equivalent 3-OPP graph for the example in Figure 2



Figure 9: Two fishbone layout designs with differences from the example

transformation is similar to the last two steps in the proposed transformation, with the exception of creating the artificial nodes  $m_{10}$  and  $m_{2n+2,0}$  on the left- and right-most corners of the middle aisle respectively.

It should be mentioned here that a similar transformation can also convert a flying-V warehouse graph into an equivalent 3-OPP graph. All that needs to be done in that case is to separate the middle aisle from the front cross aisle by creating a copy of the P&D node, transforming the middle aisle to a straight line by keeping the distances on the middle aisle same as the original graph.

Based on the validity and tractability of the transformation as well as its applicability to flying-V layouts, and polynomial time solvability of 3-OPP on parallel-aisle warehouse graphs due to Roodbergen and De Koster [12], we can now state the main theorem of this section.

**Theorem 1** *The OPP in fishbone and flying-V warehouses can be solved in polynomial time.*

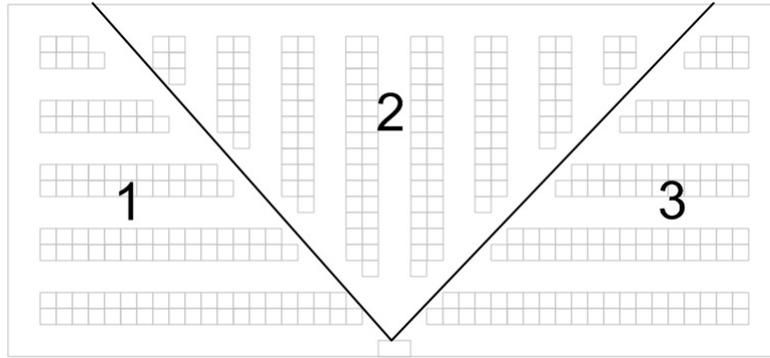


Figure 10: Division of the fishbone layout into regions for application of the heuristics

Compared to the traditional 3-OPP graph with parallel pick aisles and a straight middle aisle, the resulting 3-OPP graph after the aforementioned transformation differs in terms of the fact that the distances between pick aisles are not identical, and pick aisle lengths are not the same throughout the warehouse. However, the algorithm that solves 3-OPP does not depend on any of these two factors, which secures the applicability of the algorithm for this case.

## 4 Heuristic approaches

While the algorithm by Roodbergen and De Koster [12] can be used to solve the OPP in fishbone layouts to optimality, its enumerative approach makes it hard to implement in practice. To overcome this problem, we propose heuristic procedures, inspired by those put forward for the traditional layouts by Hall [7], Vaughan and Petersen [15], and Roodbergen and De Koster [13].

One idea to apply such heuristics is to transform the fishbone graph into an equivalent 3-OPP graph and apply the heuristics on this graph. However, based on the fact that pick aisles are no longer of equal length, this results in excessive travel in all of the heuristics. Instead of directly applying the heuristics on the resulting 3-OPP, we will divide the original warehouse graph into regions, treat these regions separately as single blocks and apply the heuristics accordingly. One such division that seems to work well with the heuristics used in this study is given in Figure 10, which was also used by Djukić and Opetuk [4].

The first heuristic we consider is the S-shape heuristic. This procedure is also applied by Djukić and Opetuk [4] using the same division structure given in Figure 10. Starting from region 1 and treating it as

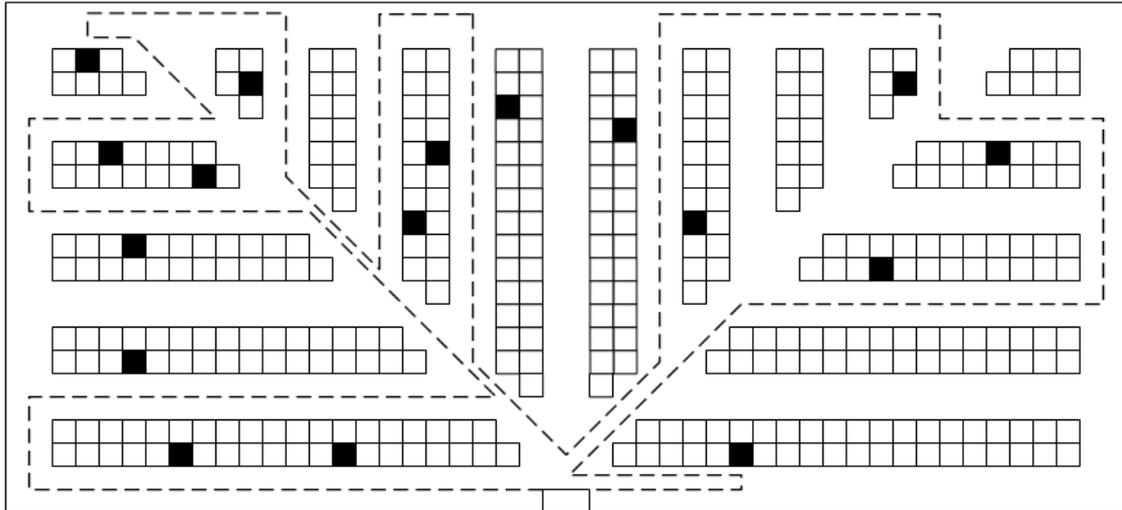


Figure 11: S-shape heuristic solution for the instance given in Figure 2

a single block, the S-shape heuristic for the 2-OPP is applied to it. When items on the back-most subaisle are picked, the picker proceeds by picking the items in the second region using the S-shape heuristic from the left-most pick aisle to the right-most one. Lastly, items in the third region are picked from the back-most to the front-most pick aisle in the same manner and the picker returns to the depot. To exemplify this procedure, Figure 11 gives the S-shape heuristic solution for the instance given in Figure 2.

Application of the largest gap heuristic is somewhat different from that of the S-shape heuristic, in that we do not apply the heuristic in sequence of the regions. Treating each region as a single block, largest gaps are found out for each pick subaisle, which determine from which corner each subaisle will be entered and exited, as was explained for the 2-OPP aisle-by-aisle heuristic. Based on these, the picker starts by going through the diagonal middle aisle on the left half, enters subaisles in region 1 from the right corner whenever needed, and enters subaisles in region 2 from the front corner whenever needed. Then, the picker proceeds to the back cross aisle and picks items in region 2 entering and exiting from the back corner whenever needed. When this is complete, the next step is to traverse the diagonal middle aisle on the right half, picking the remaining items in region 2 by entering and exiting from the front corner, and items to be picked by entering and exiting from the left end in region 3. Upon completing this, the picker proceeds to the right end of region 3 and picks the remaining items entering and exiting from there. Lastly, the picker moves to the left end of region 1 from the back cross aisle, and picks the remaining items entering and exiting from the left end, and returns to the depot. The largest gap heuristic solution for the instance in Figure 2 is given in Figure 12.

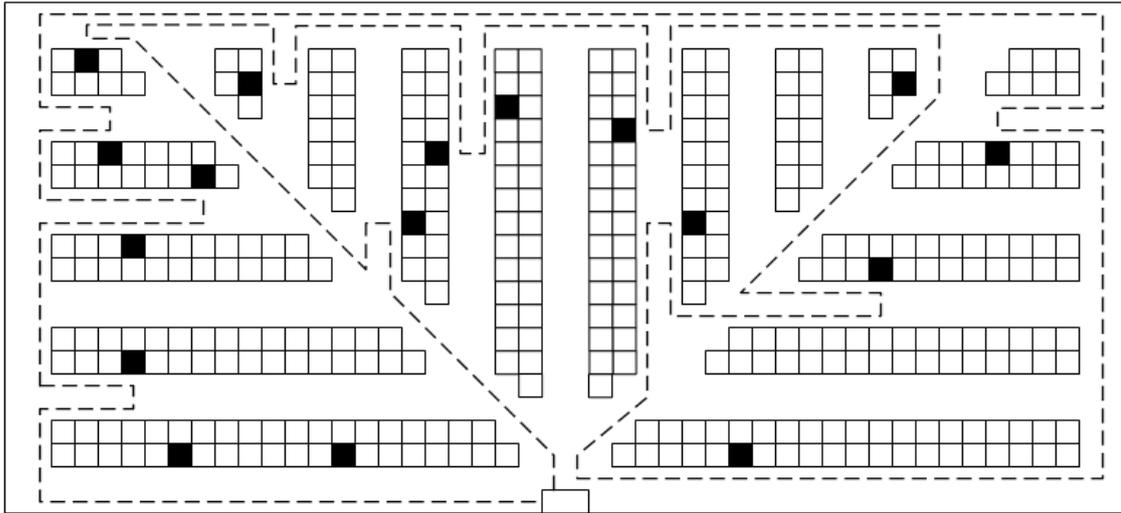


Figure 12: Largest gap heuristic solution for the instance given in Figure 2

It can also be observed from Figure 12 that the largest gap procedure results in excessive travel due to the fact that the back cross aisle is traversed twice. Since the picker has to enter each block from two ends, he has to travel the middle aisle, the left- and right-most aisles as well as the front and back cross aisle completely. Figure 13 shows these in a graphical way. It can be observed from the figure that this results in a semi-Eulerian graph, which means double traversal of at least one of these aisles will be unavoidable. The cheapest conversion to a Eulerian graph can be seen to be by traversal of the back cross aisle twice, resulting in the proposed largest gap procedure.

The aisle-by-aisle heuristic, which is identical to applying the combined heuristic as we consider regions as single block layouts with no middle aisle, is applied in a similar way to the S-shape heuristic. The picker proceeds sequentially region 1 through region 3, applying the DP-based aisle-by-aisle heuristic for each one. In Figure 14, the aisle-by-aisle heuristic result for the problem instance in Figure 2 is given.

Before ending this section, it should be noted here that the division procedure discussed here is not necessarily the one that gives the best route given the selected heuristic procedure. However, better division procedures will be complicated and hence harder to implement in practice. Hence ease of application favors the aforementioned division procedure over possibly better ones.



## 5 Computational Experiments

In this section, we describe our computational experiments, aimed at achieving two objectives. First, we would like to see how fishbone layouts compare to equivalent parallel-aisle layouts under optimal routing of pickers when multiple-item pick lists are present. Secondly, we would like to compare the relative performances of the heuristics proposed in this study with regard to the optimal routing.

While proposing optimal fishbone layouts under single and dual command operations, Gue and Meller [5] have found out the best possible improvement of the fishbone layout over the traditional one as 23.5% for single command operations, and Pohl et al. [9] have observed the maximum improvement of the fishbone layout over traditional layout with no middle aisle as 15%, and over one with a middle aisle as 10%, when dual command operations are considered. As the number of lines in the pick-list increases, the performance of the fishbone layout is expected to deteriorate. To analyze the extent at which traditional layout performs better than fishbone layout in the existence of larger pick lists and determine the order size at which traditional layout starts to perform better, in line with the computational experiments of Roodbergen and De Koster [13], we test the performance of fishbone layout on a number of randomly generated instances, where 2,000 instances are generated for each setting to estimate the mean travel time within 1% relative error with probability 95%. For each of the instances in the experiments, the width of the cross (and middle) aisles is set to 2.5 m., as are the distances between each aisle. The walking speed of the picker is set at 0.6 m/s. The experiments have been carried out by varying different parameters of the traditional warehouse with a middle aisle and converting each to a fishbone warehouse with the same number of pick aisles and same total pick aisle length, using a reversed version of the transformation procedure described in the preceding section.

Within the scope of this study, we employ a descriptive approach in terms of the fishbone layout rather than a prescriptive one, in that instead of finding an optimal layout for single or dual command operations minimizing expected travel time, we assume a fishbone layout for which the middle aisle ends at the corners of the back cross aisle. We also assume that the ends of subaisles meet on the middle aisle of the fishbone layout. Since the number of aisles and the width between the aisles are fixed for each instance, the total width of the warehouse is fixed. Due to this, to investigate the size of the warehouse, different depth/width ratios are used. The parameter settings for the traditional warehouse are summarized in Table 1.

Table 1: Problem parameters used for the computational experiments

Number of aisles	7 and 15
Number of items	1, 2, 3, 4, 5, 10, and 30
Depth/width ratio	1/3, 1/2, 2/3, 1, 2, and 3

Table 2: Experimental results for the percent gap between the average travel times for traditional and fishbone layouts

		Depth/Width Ratio					
Number of aisles	Number of items	1/3	1/2	2/3	1	2	3
7	1	-13.01	-13.29	-15.41	-17.47	-18.12	-19.56
	2	-5.20	-6.43	-7.54	-9.76	-10.30	-11.41
	3	2.37	2.01	1.13	0.64	2.16	3.48
	4	3.59	5.31	7.18	9.21	9.77	10.86
	5	6.56	6.24	7.94	8.39	10.70	15.12
	10	7.82	9.12	12.56	16.06	21.11	25.92
	30	9.67	12.40	15.10	20.31	26.35	30.61
15	1	-14.52	-15.61	-15.82	-18.35	-18.82	-20.80
	2	-2.95	-3.38	-5.94	-8.22	-9.96	-12.95
	3	2.27	2.41	0.79	1.53	1.82	3.61
	4	3.83	4.88	7.10	8.36	10.47	11.93
	5	8.23	9.08	9.48	10.79	11.16	16.72
	10	9.89	13.21	17.83	20.59	26.71	32.10
	30	13.01	16.62	19.34	23.96	29.44	36.26

With two different settings for the number of aisles, seven different settings for the number of items, six different settings for the depth/width ratio, and solving 2,000 instances for each setting, we solve a total of 168,000 instances. For each instance, the distance matrices are calculated and the optimal solutions are obtained using the Concorde [3] TSP solver. For each instance, the gap is calculated as:

$$Gap = \frac{z^{FIS} - z^{TRA}}{z^{TRA}},$$

where  $z^{FIS}$  refers to the travel time for the fishbone layout, and  $z^{TRA}$  represents the travel time for the traditional layout. Table 2 summarizes the results of the computational experiments by indicating the percent gap between the average travel time for the 2,000 instances for each setting, between the traditional and fishbone layouts. Due to the setting of the gap, it may take negative values, which indicates that the fishbone layout performs better than the traditional one in that setting.

The first inference that can be drawn from Table 2 is the superiority of the fishbone layout over the traditional one for a single item (single command) and two items (dual command), which is in line with the

findings of Gue and Meller, and Pohl et al. The maximum improvements of 20.80% for the single command, and 11.95% for the dual command are close to the theoretical bounds of 23.5% and 10-15% respectively. The small differences can be attributed to fact that the fishbone layouts may not be optimally designed. As the number of the items in the pick list is increased, the good performance of the fishbone layout diminishes, as expected. Its performance can be overrun by the traditional layout at an average rate of 36.26%, when the size of the pick-list is 30.

Another factor that contributes to the quality of the fishbone layout solutions is depth-to-width ratio of the warehouse. As this number is small (which is the case for most traditional warehouses, as it has superior travel distances over the high ones), the traditional and fishbone layouts are expected to perform similarly, as the “middle aisle” of the fishbone layout acts almost like a middle aisle for the traditional layout. The results in Table 2 confirm this expectation. For 1- or 2-item pick-lists, the gap is a negative number closer to zero than that of higher depth/width ratios. For larger pick-lists, the gap is a small number for small ratios, and larger as the ratio gets larger.

As the number of aisles increases, one would expect to see a greater difference between the performances of the fishbone and traditional layouts, as for small pick-lists, the distance between the depot and the items would be higher for traditional warehouses with increasing number of aisles. Similarly, for the case of large pick-lists, the distance between the items, which would be the dominant factor in the total travel time, would be higher for warehouses with higher number of aisles. This is also verified by Table 2, where the absolute value of the gap is higher for the 15-aisle instances.

To test the relative performances of these heuristics, we use the same problem set in the previous section, whose optimal solutions are readily available. Table 3 gives the percent optimality gaps for these heuristics for various problem settings.

One observation that can be inferred from these results is that the problem settings do not have a significant effect on the performance of the heuristics. Comparing the heuristics between themselves, it can be seen that aisle-by-aisle (with a maximum deviation of about 21%) outperforms S-shape (with a maximum deviation of around 26%) by a slight margin, whereas both of these heuristics significantly outperform largest gap (whose maximum deviation is more than 37%). The performance results for the S-shape heuristic are close to those obtained by Djukić and Opetuk [4]. The main reason behind why largest gap does not perform well

Table 3: Percent optimality gaps of the heuristics under various problem settings

			Depth/Width Ratio					
Heuristic	No. of Aisles	No. of Items	1/3	1/2	2/3	1	2	3
S-shape	7	10	25.66	24.83	22.36	15.54	13.40	12.26
		30	18.87	16.89	14.92	13.49	11.73	9.76
	15	10	22.22	19.74	16.56	14.07	13.61	11.04
		30	26.10	24.12	22.09	18.81	16.71	14.09
Largest gap	7	10	31.27	29.10	26.30	25.69	23.41	22.90
		30	31.40	30.16	28.06	26.71	22.50	19.51
	15	10	37.73	35.46	33.82	29.91	27.66	24.46
		30	36.29	33.90	30.62	27.27	23.74	20.15
Aisle-by-aisle	7	10	20.78	19.09	16.83	15.43	12.49	11.63
		30	15.01	13.52	11.24	9.92	9.46	8.88
	15	10	17.62	16.30	15.96	13.09	12.82	9.63
		30	20.12	17.15	16.53	14.98	13.82	11.25

is the fact that the fishbone graph structure may not be suitable for its application due to excessive travel on the back cross aisle (which is unavoidable due to the current division structure) changing length of the pick aisles.

## 6 Conclusions and Further Research Directions

Fishbone layouts, first proposed by Gue and Meller [5], improve the performance of the warehouse layouts in the case of a small number of items in the pick-list. In this paper, we have tried to determine the performance of the fishbone layout in the existence of larger pick-lists. For this end, we have first proven that the order-picking problem can be polynomially solved on a fishbone warehouse graph. Then, we have implemented the fishbone design on randomly generated problem instances to observe the performances. We have seen that the fishbone layout outperforms the traditional one with a good average improvement, and the maximum improvement is close to the theoretical bound set by Gue and Meller [5]. As the size of the pick-list grows larger, it is outperformed by the traditional layout, with a maximum gap of around 36%. Lastly we have adapted a number of heuristics for the traditional warehouses for order picking in fishbone layouts under pick-lists. Based on our computational results, we have observed that the aisle-by-aisle heuristic outperforms its counterparts, with an average deviation no worse than 21%.

For a considerable number of warehouses, the demand distribution for the items in the warehouse is rarely random. Usually, items with higher demand are placed in closer locations to the depot point. Hence one extension of the analysis conducted in this paper would be the relaxation of the non-uniform assumption for the item orders. Another extension would be to investigate the theoretical bounds for average (as opposed to maximum) improvement in the case of uniform demand, and average and maximum bounds on the improvement for more realistic distributions of demand locations.

A better performance measurement of the fishbone layout compared to the traditional one under optimal routing would be to consider an optimal design that minimizes expected travel time under a given pick list size. This is not only difficult to achieve, but is also not realistic as pick list size changes with orders. One way to overcome this is to assume an average pick list size, in which case calculation of the expected travel time would be a big challenge. In any case, we need to note that the gaps obtained in this study may underestimate the relative performance of the fishbone design, although not very significantly.

Lastly, the application of heuristics requires a division of the warehouse into blocks, for which only one specific way is tested. More complicated divisions that decrease the average travel time would also be interesting to analyze.

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