CONTINUOUS APPROXIMATION OF MULTI CYCLE TIME FOR MULTI AISLES AUTOMATED STORAGE AND RETRIEVAL SYSTEMS.

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Abstract

Knowing that, in an automated storage and retrieval system, the travel time of the storage/retrieval machine is an important parameter which affects the whole system performances, several works were dedicated to its evaluation and its modeling, in single, dual and multi command. These models were proposed for different configurations of AS/RS; such as unit load, multi-aisles and Mobile Racks. These models are based mainly on a discrete approach and the analytical expressions represent the system functioning exactly.
In this paper we present a general approach for analytical modeling of multi command cycle time by giving particularly the modeling of the Time Between that we will use to model the Dual Cycle time and the Multi Cycle time. This continuous approach with its simplified mathematical expressions aims to make the calculations more easy than the discrete approach and can be generalized for any one deep physical configuration of AS/RSs and later used to make an optimization of the dimensions of such systems.

Key words: Multi-aisles automated storage and retrieval systems, analytical modeling, multi-cycle.

1 Introduction:

One of the most important goals for a Flexible Manufacturing System (FMS) is to maximally reduce the activities that could, uselessly, increase the costs. One of these activities, and not the least, is storage [1]. Indeed, the storage of raw materials, tools, work in process and finished products costs time, money and space without adding any value to the stored products [1].

For FMSs, several configurations of automated storage/retrieval systems were developed and used years ago. Till now, they evolve in consideration of different parameters : permanent need of space for storage, variety in size and kind of stored products, the major need of optimizing storage costs (time, space, money, scheduling). AS/RSs are combination of automatic equipments [2] organized depending to the system configuration (Racks, S/R machines, Aisles, Pick-up/Deposit stations, Control system) and have lot of advantages. The conception of an AS/RS need to consider some influencing parameters according to their use such as dimensions and number of bins, number of S/R machines, duration of cycle time (single, dual, multi), use rate [3] & [4] and are generally classified in various types according to their physical configurations such as the number and capacity of S/R machines.

In basic configurations, the S/R machine have a unit-load capacity and could do only single cycles or dual cycles, but in more general configurations, the S/R machine could have a multi-load capacity and could do multiple cycles (multi command).

In this paper, we are interested in one-deep configuration of AS/RSs such as Unit Load AS/RS (UL-AS/RS), Mobile Racks AS/RS (MR-AS/RS) or Multi Aisles AS/RS (MA-AS/RS), and we aim to present a general approach based on a continuous approximation in order to model the travel time of a multi command cycle, by calculating the Time Between travel time, and we'll show how we can generalize this approach to different physical configurations for one-deep class AS/RSs and use the resulting expressions for a future work dedicated to the optimization of the dimensions of AS/RSs.

So, in section 2 we give a summarized state of art about works already done on AS/RSs and travel time modeling. In section 3 we present a general description of one-deep AS/RSs and details on operating modes of S/R machines, basic moves and related cycles times. Section 4 is devoted to presentation of the approach we developed with its analytical expressions to model the cycle time of S/R machine in several operating modes. Finally, we conclude the paper and present perspectives of our work.

2 State of art:

In this section we give a brief general overview about researches previously done on existing one-depth AS/RSs and the modeling of S/R machine's travel time (storage/retrieval time).

AS/RS have been the subject of many studies years ago. A detailed survey on AS/RSs design, classification and functioning policies is given by Roodbergen in [13]. Modeling of the average travel time has been developed in numerous studies since 1976.

Hausman & *al.* [14] modeled with continuous expressions, the single cycle travel time for a square-in-time unit-load AS/RS assuming that the rack is square-in-time and using various storage policies.

Bozer & White in [5] gave analytical models of single and dual command cycle time for a non square-in-time UL-AS/RS. They considered random locations of the P/D station and storage rules.

In [15] Hwang and Lee said that previous studies model's were not optimal because their authors considered S/R machines having a uniform velocity. Therefore, they presented their model based on consideration of the S/R machines operating characteristics (acceleration, deceleration and maximum speed) and non square-in-time racks. Chang & *al.* [16] suggested a travel time model of an S/R machine, with several speeds, where the acceleration and deceleration were taken into account.

MA-AS/RS is almost similar to a UL-AS/RS, at the difference of, in a MA-AS/RS all aisles are deserved by only one S/R machine. Several researchers devoted works to model the travel time in a MA-AS/RS. Hwang and Ko [7] proposed a single cycle travel time model for a MA-AS/RS, starting from the model proposed by Bozer & White [5] for a UL-AS/RS. Lerher & *al.* [12] model considered the acceleration and deceleration of the S/R machine. Ghomri & *al*. in [8] developed a model of average single cycle travel time for a MA-AS/RS. All these authors suggested the surface of a rack as a continuous plane and approximated it to calculate the total travel time which equals the average of travel times of all racks, except [8] where the system was considered as a continuous right-angled parallelepiped, and resulted a purely continuous formula, which was used by Kouloughli & *al.* [17] to optimize multi-aisle AS/RS form.

3 One-Depth Automated Storage and Retrieval Systems :

Automated storage and retrieval systems (AS/RS) are major material handling systems, which are widely used in automated productions and distribution centers [12] for putting products (e.g., raw materials or semi-finished products) in storage and for retrieving them from storage to fulfill an order[13]. An AS/RS is capable of handling pallets without the interference of an operator, simply using a set of automated components. The common basic components of AS/RS are racks, bins, storage and retrieval machines (S/R machines), Pickup/Deposit stations (P/D stations), conveyors.

AS/RSs have various advantages: efficient use of warehouse space, reducing damages and lost goods, increased control upon storage and retrieval, decreasing the number of warehouse workers. But AS/RS are rather expensive and inflexible in future changes [12]. The choice of S/R machines number is closely depending on the throughput capacity. For a high capacity a single machine into the single picking aisle is recommended. For low capacity, double deep AS/RS can be used [12].

3.1 Physical configuration :

Figure 1 represents the physical configuration of a One Depth AS/RS and describes its different composing elements. In a UL-AS/RS there is a dedicated S/R machine to each aisle. In case where only one S/R machine deserves all aisles of the system this configuration is called MA-AS/RS. We also give the notation used to describe the functioning of these AS/RS.

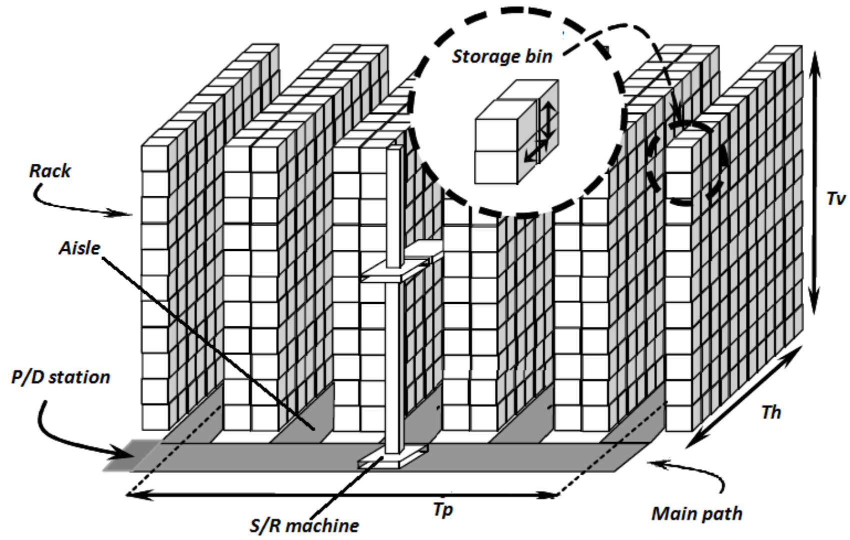


Figure 1 : Physical configuration of a One Depth AS/RS

3.2 S/R machine functioning :

As said above, the S/R machine is used to store and retrieve items in and from random bins situated in racks. In case of MA-AS/RS these bins could be situated in various aisles.

- *Single command mode* : the basic work of S/R machine is to start from the P/D station taking an item to store it in a bin and come back to this same P/D station. This move is called single command mode where the machine is used to do only one job (store or retrieve) at a time.
- *Dual command mode* : an other working mode exists, where the S/R machine starts from the P/D station taking an item to store in a bin, after moves to an other bin which could be situated in a different aisle, to retrieve an other item and finally go back to the P/D station. This is the dual command mode where the machine is used to store and retrieve items at the same time. This mode is useful to eliminate "hand free" come backs of S/R machine to the P/D station.
- *Multi command mode :* the latest functioning mode is the multi command mode, where the S/R machine is used to do multiple storing and retrieving tasks during one go/come back to the P/D station. This mode suppose that the machine has a certain multi-loads capacity and which is useful to minimize go/come back travels from/to the P/D station and optimize the use of the S/R machine.

3.3 Basic moves and travel times :

The S/R machine is an automated machine commanded by 3 computer-controlled motors allowing the machine to move in horizontal and vertical directions when reaching any bin in the system. Indeed, according to figure1 the machine moves : horizontally along a rack length and vertically along the a rack height to reach a bin in a UL-AS/RS configuration.

In other configurations as MA-AS/RS or MR-AS/RS, the machine moves also horizontally on the main path to change aisle.

- Along rack length : S/R machine moves horizontally along the length of a rack to move from a bin to another, during a certain horizontal travel time noted *Th;*
- Along rack height : S/R machine moves vertically throw the height of a rack to move from a bin to another, during a certain vertical travel time noted *Tv;*
- Main path : S/R machine parcourate horizontally along the main path to move from an aisle to another, during a certain horizontal travel time noted *Tp.*

According to figure 1 we have the following notation :

Tp : Travel time necessary to reach an aisle along the main path starting from P/D station.
Th : Travel time necessary to reach a bin along the length of a rack.
Tv : Travel time necessary to reach a bin throw the height of a rack.
E(SC) : Expected Single Cycle time
E(DC) : Expected Dual Cycle time
E(TB) : Expected Time Between (time necessary to travel between 2 random bins).

In order to economize displacement time the machine can move simultaneously in both horizontal and vertical directions. This is called Tchebychev move.

5 Cycles Times modeling :

Investing in AS/RS has significant costs. This is why their conception need to consider many physical design (dimensions, components, ...) and control (configuration, times, …) issues in order to use them in an optimal way and to fully benefit of their advantages. More details on designing decisions can be found in [13].

The S/R machine only represent 40% or more of initial costs of the whole system [12], and thus, one of the most important parameters in designing AS/RS and optimizing its performances is the travel time of this machine. For this reason, we focus in our paper on the modeling of such a travel time, knowing that it could bring positive and significant advantages in terms of efficiency and accuracy, and consequently throughput and productivity.

In this paper we will take the MA-AS/RS configuration as an example for applying our approach and to show calculation processing and expected results.

5.1 Multi-Aisles AS/RS (MA-AS/RS) description :

One of the most studied form of AS/RS is the Multi-Aisles Automated Storage and Retrieval System (MA-AS/RS), which is composed of : several parallel aisles, each aisle has one rack on each side, each rack contains a certain number of bins. The whole system is served by only one S/R machine which works in modes previously described : single command, dual command and multiple command.

System characteristics :
– The AS/RS contains several parallel aisles and each aisle has a storing rack on both sides and each rack contains several bins.
- P/D station is located at point 0 at the start point of the main path and the bottom of the first rack.
– The S/R machine start point for each cycle is P/D station and travels to any random bin throw main path and aisles. It operates in Single, Dual Command Cycle and may operates in Multi Command.
– The S/R machine has a Tchebychev move i.e it moves simultaneously in the horizontal and vertical directions.
– Racks and aisles are considered as continuous surfaces.

A cycle time is defined as the time taken by the S/R machine to go from the P/D station, do a task and come back to P/D station. Do a task could be :

- Storing in a bin (Single command = Single cycle time)
- Retrieving from a bin (Single command = Single cycle time)
- Storing + Retrieving (Dual command = Dual cycle time)
- [Storing + Retrieving] several times (Multi command = Multi cycle time)

So, whatever will be the functioning mode, every move of the S/R machine from a point to another, will be a set of 3 simultaneous basic moves : the horizontal move on main path to reach an aisle K during the time *Tp*, the horizontal move along the length of a rack to reach a point X during the time *Th* and the vertical move along the height of a rack to reach a point Y during the tie *Tv.*

Due to the Tchebychev move, we consider that a cycle time is composed of 2 kind of times : horizontal travel time (*Th* + *Tp*) and vertical travel time (*Tv*).

5.1 Single Cycle time :

In a Single cycle, the machine do only one task (store or retrieve) starting from P/D station and coming back to it. The physical travel is illustrated with figure 2 while the travel time is represented by figure 3.

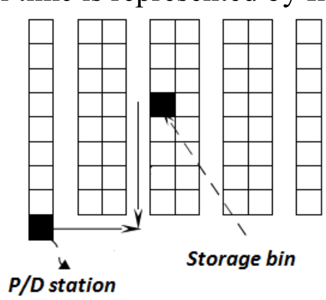


Figure 2

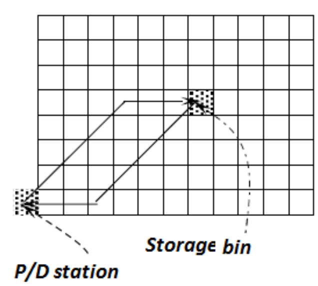


Figure 3

5.2 Dual Cycle time :

In a Dual cycle, the machine do two consecutive tasks : go to store in a bin with coordinates (x1,y1) in aisle k1 starting from P/D station, after go to retrieve from another bin with coordinates (x2,y2) in aisle k2, before coming back to P/D station. The physical travel is illustrated with figure 4 while the travel time is represented by figure 5.

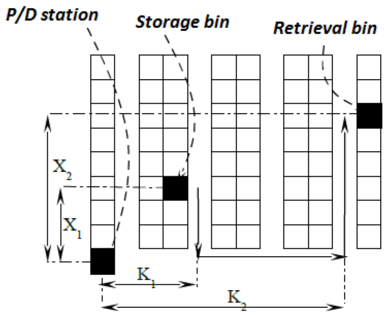


Figure 4

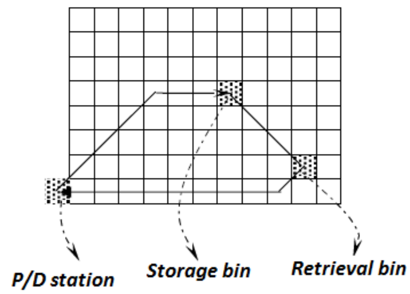


Figure 5

In this cycle there a certain time taken by the machine to move from the first storing bin (x1,y1) in aisle k1 to the second retrieving bin (x2,y2) in aisle k2. This time is called Time Between E(TB), and this move is also composed of horizontal and vertical moves and times as mentioned in section 5.1. This E(TB) is the main idea we focus on for the next steps of our work.

6 Continuous approximation for modeling cycle times :

In our modeling approach of cycles times with a continuous approximation we consider a rack as a continuous surface where a bin has coordinates (x,y). As we see in figure 6, reaching a bin in a rack is reaching coordinates (x,y) on the continuous surface. (X axe's corresponds to *Th* and Y axe's to *Tv*). Our approach is different from [6] and [7] because it is purely continuous and we use purely continuous expressions.

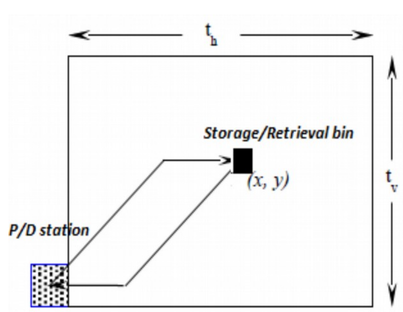


Figure 6

Due to the limitation of the paper pages, we could not give details of all calculations because of their length and the similarity in calculations for all the possible cases, but we will detail only one case and present directly the results for remaining cases.

6.1 Single Cycle time E(SC) :

The single cycle time E(SC) is composed of a horizontal travel time $SC_h = (Th+Tp)$ and a vertical travel time $SC_v = Tv$. The S/R travel time to reach bin (x,y) equals to the maximum between SC_h and SC_v, and so the total E(SC) to go and come back is:

$$E(SC) = 2*\mathrm{Max}(SC_h, SC_v) = 2*\mathrm{Max}(Th+Tp, Tv).$$

Th and *Tp* are durations of time and so they are random discrete variables with uniform distribution which we approximate with continuous uniform distributions. Figures 7 & 8 are respectively the exact discrete distribution functions for *Tp* and *Th.* Figures 9&10 are the approximated continuous distribution functions for *Tp* and *Th*.

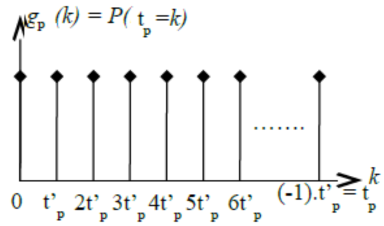


Figure 7

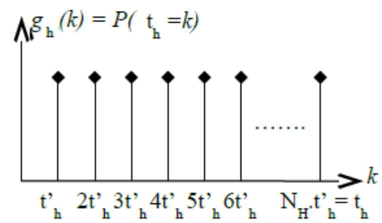


Figure 8

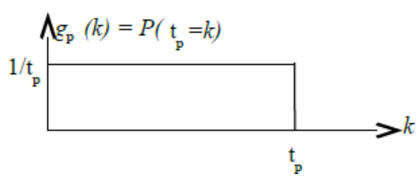


Figure 9

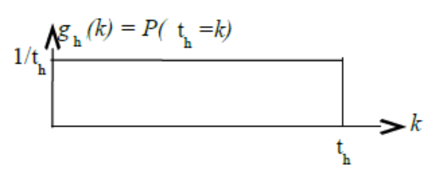


Figure 10

The horizontal time distribution function for *Th* and *Tp* are given by :

$$g_h(k) = \begin{cases} \dfrac{1}{t_h} & 0 < k < t_h \\ 0 & \text{otherwise} \end{cases} \qquad g_p(k) = \begin{cases} \dfrac{1}{t_p} & 0 < k < t_p \\ 0 & \text{otherwise} \end{cases}$$

Since the horizontal travel time is the sum of *Th* and *Tp,* then its probability density function *g(k)* equals to the convolution product of *Th* and *Tp* functions:

$$g(k) = g_p(k) * g_h(k) = \int_{-\infty}^{+\infty} g_p(x) * g_h(x-k)dx = \int_0^x g_p(x) * g_h(x-k)dx$$

After calculation of this convolution product we obtain the following equation and its representative scheme in figure 11 :

$$g(k) = \begin{cases} \dfrac{k}{tp\ th} & k < \min(tp, th) \\ \dfrac{1}{\max(tp, th)} & k < \max(tp, th) \\ \dfrac{tp + th - t}{tp\ th} & k < tp + th \\ 0 & otherwise \end{cases}$$

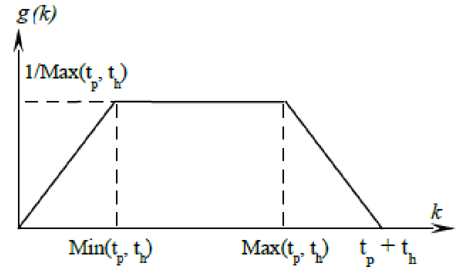


Figure 11

The vertical time distribution function for *Tv* is given by :

$$h\ (k) = \begin{cases} \dfrac{1}{tv} & k < tv \\ 0 & otherwise \end{cases}$$

Knowing that the total travel time of the S/R machine is the maximum between the horizontal and vertical times (to go + to come back) :

*E(SC)= 2*Max(Th+Tp, Tv)= 2*Max(*SC_h*,* SC_v*) =* $2 * \int_{-\infty}^{+\infty} k * f(k)dk$

its distribution function *f(k)* is :

$$f(k)=G(k).h(k)+H(k).g(k)$$

where :

$$G(k) = P(SC_h < k) = \int_{-\infty}^{k} g(t)dt$$

is the repartition function of the horizontal travel time

and

$$H(k) = P(SC_v < k) = \int_{-\infty}^{k} h(t)dt$$

the repartition function of the vertical travel time.

During calculation of the time distribution of the machine E(SC) we'll face 4 cases represented by figure 12 :

- Case 1 : *Tv*<Min(*Tp*,*Th*)
- Case 2 : Min(*Tp*,*Th*)<*Tv*<Max(*Tp*,*Th*)
- Case 3 : Max(*Tp*,*Th*)<*Tv*<*Tp*+*Th*
- Case 4 : *Tv*>*Tp*+*Th*

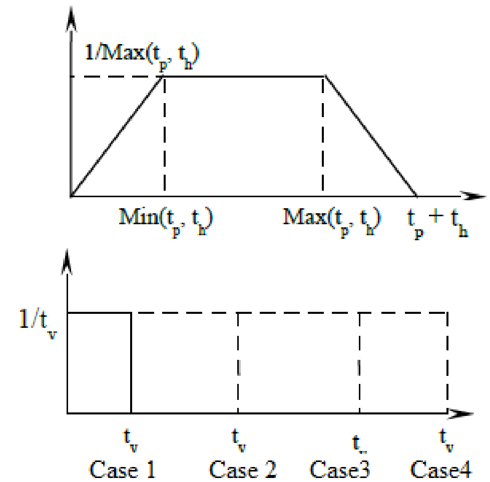


Figure 12

For each case we obtain following results for E(SC) :

$$
ESC = \begin{cases}
tp + th + \dfrac{tv^3}{12\ tp\ th} & 0 < tv < \min(tp, th) \\[2ex]
tp + th + \dfrac{tv^3}{12\ tp\ th} - \dfrac{(tv - \min(tp, th))^4}{12\ tp\ th\ tv} & \min(tp, th) < tv < \max(tp, th) \\[2ex]
tp + th + \dfrac{tv^3}{12\ tp\ th} - \dfrac{(tv - \min(tp, th))^4 + (tv - \max(tp, th))^4}{12\ tp\ th\ tv} & \max(tp, th) < tv < tp + th \\[2ex]
tp + th + \dfrac{tv^3}{12\ tp\ th} + \dfrac{(tv - \min(tp, th) - \max(tp, th))^4 - (tv - \min(tp, th))^4 + (tv - \max(tp, th))^4}{12\ tp\ th\ tv} & otherwise
\end{cases}
$$

We use the Heavyside function HS(t) defined below to simplify previous equation ESC.

$$
HS(t) = \begin{cases} 0 & t < 0 \\ 1 & otherwise \end{cases}
$$

So consequently : $(t-u).HS(t-u)=Max(t-u,\ 0)=Max(t,u)-u$ **(a)**

Using function HS(t) we can write equation ESC as following:

$$
\overline{ESC} = tp + th + \frac{tv^3}{12\ tp\ th} + \frac{(tv - \min(tp, th) - \max(tp, th))^4 . HS(tv - \min(tp, th) - \max(tp, th)) - (tv - \min(tp, th))^4 . HS(tv - \min(tp, th)) - (tv - \max(tp, th))^4 . HS(tv - \max(tp, th))}{12\ tp\ th\ tv}
$$

and using **(a)** we can rewrite ESC to obtain the final and easily calculable expression of the mean travel time of the S/R machine in Single Command cycle as following:

$$\overline{E(SC)} = \frac{tp + th}{4} + \frac{tv}{2} + \frac{tv^3}{12\,tp\,th} + \frac{2\,(tp+th)^2 - tp\,th}{6\,tv} + \frac{(tv - tp - th)^3 \max(tv,\ tp+th) - (tv - th)^3 \max(tv, th) - (tv - tp)^3 \max(tv,\ tp)}{12\,tp\,th\,tv}$$

This reasoning was followed in [2] for modeling the average SC time and here we'll do the same in order to model the average TB time.

6.2 Dual Cycle time E(DC) :

The dual command travel time as well has been modeled in [5],[6],[7].

In [6] the work is based on a discrete model of DC time which is quite heavy to calculate. In [7] the DC time is estimated by a hybrid (discrete and continuous) model and which is a bit lighter than [6] but still heavy for calculation. However, only the case where storage and retrieval bins are situated in the same aisle was treated by [5] in dual command for a UL-AS/RS which is considered as a particular configuration of an MA-AS/RS. The case of different aisles (random storage/retrieval bins) still remain untreated and this is what we aim to do hereunder.

Referring to [2], the Single Command travel time is the mean travel time to go from the P/D station to a random storage bin and come back. Figure 7 shows that.

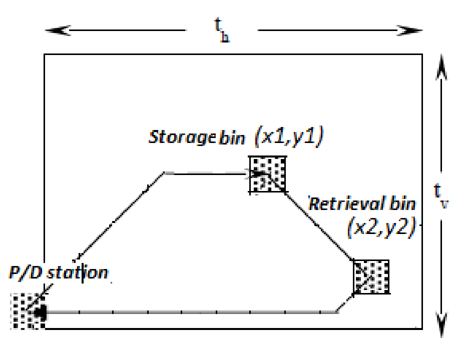


Figure 13

According to figure 13 we can see that a Dual Command travel is composed of the moves: from P/D station to storage bin, from storage bin to retrieval bin, from retrieval bin to P/D station.

This way we can conclude that a Dual Command mean travel time in a MA-AS/RS is simply the sum of travel time of a Single Command travel time plus a Time Between (TB) travel time. This TB is the time taken by the machine to travel between the first random storage bin and the second random retrieval bin. In this work, we focus on this main idea and so, we summarize this reasoning by following formula:

$$E(DC)=E(SC)+E(TB). \qquad \textbf{(1)}$$

In order to calculate (1) we need to calculate *E(SC)* using a continuous approximation and also *E(TB)*. We have already show how to do for *E(SC)* previously. Now we need only to model E(TB). Our modeling covers all randomly chosen bins. It is a purely mathematical processings which consists on approximating random discrete variables with continuous ones.

6.3 Time-Between (TB) :

We suppose that the machine stores in a bin with coordinates X1,Y1 situated in aisle K1 and retrieves from a bin with coordinates X2,Y2 situated in aisle K2. We use the same notation and we consider X1, X2, Y1, Y2, K1, K2 as durations (travels times).

By the same reasoning, E(TB) is the maximum between two values $E(TB)_H$ corresponding to (2.1) and $E(TB)_V$ corresponding to (2.2), respectively horizontal travel time (from X1 to X2 and from K1 to K2) and vertical travel time (from Y1 to Y2). This means that $E(TB) = Max\ (E(TB)_H\ ,\ E(TB)_V)$ and consequently :

$$E(TB) = \text{Max}\ (\underbrace{\underbrace{X1+X2}_{T_h}+\underbrace{|K1-K2|}_{T_p}}_{\substack{\textbf{(2.1.2)}+\textbf{(2.1.2)}\\ E(TB)_H\\ \textbf{(2.1)}}}\ ,\ \underbrace{\underbrace{|Y1-Y2|}_{T_v}}_{\substack{E(TB)_V\\ \textbf{(2.2)}}}) \qquad \textbf{(2)}$$

6.3.1 Description of the continuous approximation approach :

Our approach is based on a set of mathematical calculations. Knowing that storage and retrieval bins are randomly chosen, X1, X2, Y1, Y2, K1, K2 are consequently considered as random discrete variables having exact uniform discrete distributions which we will approximate with continuous uniform distributions.

To calculate (2) using a continuous approximation, we follow a set of steps which were also previously used to calculate E(SC) for Single Cycle time.

- Step 1: give the continuous distribution functions *HD(x)* for (2.1) and *VD(x)* for (2.2).
  - Since (2.1) is a sum of (2.1.1) and (2.1.2), the distribution function of (2.1) is the convolution product of (2.1.1)'s and (2.1.2)'s distribution functions respectively *h(x)* and *p(x).*

$$HD(x) = h(x)*p(x). \qquad \textbf{(3)}$$

- Since (2.1) is composed of two basic horizontal moves and so two horizontal travel times *Th* (X1+X2) and *Tp* (|K1-K2|), we need to consider three possible cases, that we'll present in next.

- Step 2 : calculate the continuous repartition functions *HR(x)* for (2.1) and *VR(x)* for (2.2) based on the previous continuous distribution functions.

$$HR(x) = \int_0^x HD(x)dx \qquad \textbf{(4)}$$

$$VR(x) = \int_0^x VD(x)dx \qquad \textbf{(5)}$$

- Step 3 : calculate the repartition function *B(x)* for *E(TB)* according to *HR(x), VR(x)* and the three possible cases.

$$B(x) = HR(x)*VR(x) \qquad \textbf{(6)}$$

- Step 4 : Calculate *E(TB)* for each possible case using the formula :

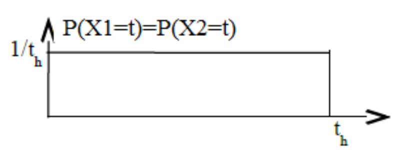
$$E(TB) = \int_0^k k * B(x)dx \qquad \textbf{(7)}$$

- Finally, *E(TB)* modeled with formula (2), mean *E(TB)* easily calculable, the calculation of *E(DC)* can be done with (1) depending on faced case.
  Next we demonstrate the analytical expressions of the approach presented above.

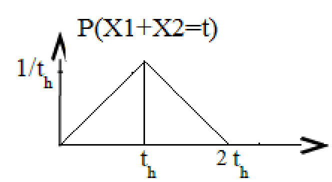
6.3.2 Analytical expressions of the continuous approximation for E(TB) :

Step 1 : Distribution functions for (2.1) and (2.2) :
X1 and X2 have the same continuous distribution. K1 and K2 also and Y1 and Y2 also.
Let start with (2.1) :

For X1+X2 :



Continuous distribution approximation for X1 and X2

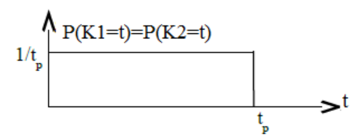


Continuous distribution approximation for X1+X2

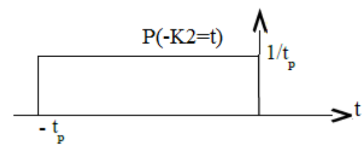
After calculation of the distribution function for X1+X2 we obtain the following result.

$$h(x) = \begin{cases} \dfrac{t}{Th^2} & 0 \le t \le Th \\ \dfrac{2\,Th - t}{Th^2} & Th \le t \le 2\,Th \\ 0 & otherwise \end{cases}$$

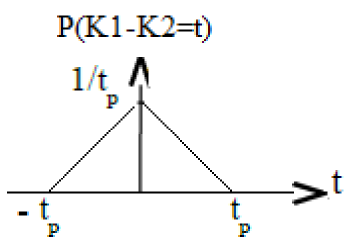
For |K1-K2| :



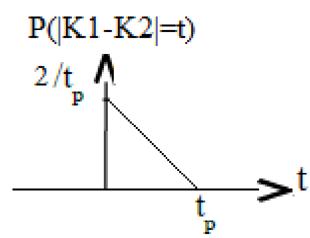
Continuous distribution approximation for K1



Continuous distribution approximation for -K2



distribution approximation for K1-K2



Continuous

Continuous distribution approximation for |K1-K2|

After calculation of the distribution function for K1-K2 we obtain the following result.

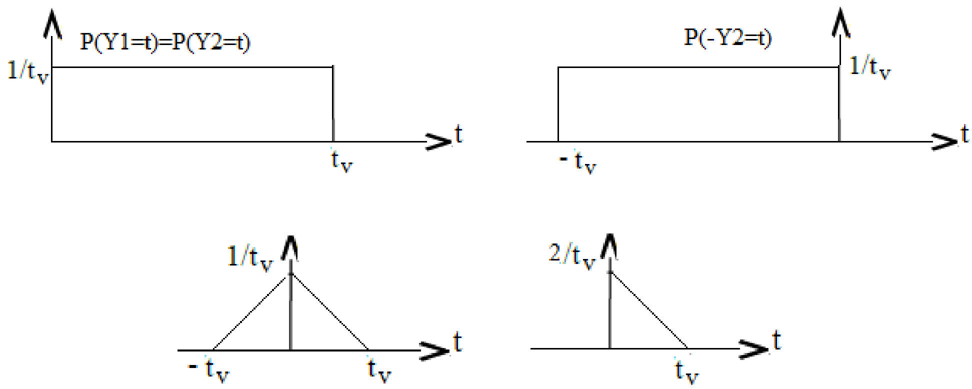
$$pp(x) = \begin{cases} \dfrac{t + Tp}{Tp^2} & -Tp \le t \le 0 \\ \dfrac{t - Tp}{Tp^2} & 0 \le t \le Tp \\ 0 & otherwise \end{cases}$$

and for |K1-K2| we have the following result :

$$p(x) = \begin{cases} \dfrac{2\,Tp - 2\,t}{Tp^2} & 0 \le t \le Tp \\ 0 & otherwise \end{cases}$$

Let continue with (2.2) :
For this part with the same reasoning we'll have the same repartition function form as

given for K1-K2, we just need to to substitute *Tp* by *Tv* and we obtain :



$$VD(x)=\begin{cases} \dfrac{2\,Tv-2\,x}{Tv^2} & 0<x<Tv \\ 0 & otherwise \end{cases}$$

Possible cases for (2.1) :
According to formula (3) and depending on the duration of *Th* and *Tv*, we will face three possible cases :

- Case (1) : $0 \le Tp \le Th$
- Case (2) : $Th \le Tp \le 2Th$
- Case (3) : $Tp \ge 2Th$

Now we give results after calculation of the horizontal distribution function *HD (x)* according to each case.

Case (1) : Tp<Th

$$HD(x)=\begin{cases} -\dfrac{x^2\,(x-3\,Tp)}{3\,Tp^2\,Th^2} & 0\le x\le Tp \\ \dfrac{-Tp+3\,x}{3\,Th^2} & Tp\le x\le Th \\ \dfrac{2\,x^3+(-6\,Tp-6\,Th)\,x^2+\left(3\,Tp^2+12\,Tp\,Th+6\,Th^2\right)x-2\,Th^3-Tp^3-6\,Tp\,Th^2}{3\,Tp^2\,Th^2} & Th\le x\le Tp+Th \\ -\dfrac{-Tp+3\,x-6\,Th}{3\,Th^2} & Tp+Th\le x\le 2\,Th \\ -\dfrac{x^3-(-3\,Tp-6\,Th)\,x^2-\left(3\,Tp^2+12\,Tp\,Th+12\,Th^2\right)x+8\,Th^3+Tp^3+6\,Tp^2\,Th+12\,Tp\,Th^2}{3\,Tp^2\,Th^2} & 2\,Th\le x\le 2\,Th+Tp \\ 0 & otherwise \end{cases}$$

Step 2 : Repartition functions for (2.1) and (2.2) :

$$HR(x)=\begin{cases} -\dfrac{k^3(k-4)}{12\,Tp^2\,Th^2} & 0<x<k \\ \dfrac{Tp^2+6k^2-4\,Tp\,k}{12\,Th^2} & 0\le x\le k \\ \dfrac{Tp^4+2k^4+2\,Th^4-8\,Tp\,k^3+8\,Tp\,Th^3-8\,Th\,k^3+6\,Tp^2k^2+24\,Tp\,Th\,k^2+12\,Th^2k^2-8\,Th^3k-4\,Tp^3k-24\,Tp\,Th^2k}{12\,Tp^2\,Th^2} & 0\le x\le k \\ -\dfrac{Tp^2+12\,Th^2+8\,Tp\,Th+6k^2-4\,Tp\,k-24\,Th\,k}{12\,Th^2} & 0\le x\le k \\ -\dfrac{24\,Tp\,Th\,k^2-48\,Tp\,Th^2k-24\,Tp^2Th\,k+12\,Tp^2Th^2+k^4+Tp^4+16\,Th^4+8\,Tp^3Th-4\,Tp\,k^3+32\,Tp\,Th^3-8\,Th\,k^3+6\,Tp^2k^2+24\,Th^2k^2-32\,Th^3k-4\,Tp^3k}{12\,Tp^2\,Th^2} & 0\le x\le k \\ 1 & \text{otherwise} \end{cases}$$

$$VR(x)=\begin{cases} -\dfrac{k^2}{Tv^2}+\dfrac{2k}{Tv} & 0<x<Tv \\ 1 & \text{otherwise} \end{cases}$$

Step 3+4 : possible values of E(TB)

Using formula (6), when calculating *B(x)* it is necessary to consider several intervals. So calculation of E(TB) according to formula (7) gives following results :

$$E(TB)1=\frac{7\,Tv^4\,Tp-Tv^5+420\,Tp^3\,Th^2+1260\,Tp^2\,Th^3}{1260\,Tp^2\,Th^2}$$

$$E(TB)2=\frac{-35\,Tv^4\,Tp-21\,Tp^3\,Tv^2+1260\,Th^3\,Tv^2+7\,Tv\,Tp^4+21\,Tv^5+35\,Tp^2\,Tv^3-Tp^5+420\,Tp\,Th^2\,Tv^2}{1260\,Tv^2\,Th^2}$$

$$E(TB)3=-\frac{\begin{gathered}-420\,Tp^3\,Th^2\,Tv^2-1260\,Tp^2\,Th^3\,Tv^2-7\,Tv\,Tp^6+2\,Th^7+Tp^7-2\,Tv^7-35\,Tv^3\,Tp^4\\+14\,Tp\,Th^6-14\,Tv\,Th^6+70\,Th^3\,Tv^4+35\,Tv^4\,Tp^3+21\,Tv^2\,Tp^5-21\,Tv^5\,Tp^2\\+14\,Th\,Tv^6+14\,Tv^6\,Tp-280\,Tp\,Th^3\,Tv^3-84\,Tp\,Th^5\,Tv-42\,Th^2\,Tv^5\\+210\,Tp\,Th^2\,Tv^4+210\,Tp\,Th^4\,Tv^2-84\,Tv^5\,Tp\,Th+42\,Tv^2\,Th^5-70\,Th^4\,Tv^3\end{gathered}}{1260\,Tv^2\,Th^2\,Tp^2}$$

$$E(TB)4=\frac{\begin{gathered}35\,Tv^4\,T\ldots\\+Tp^5+84\ldots\\+210\,Th\,T\ldots\end{gathered}}{\ldots}$$

$$E(TB)_5=-\frac{\begin{gathered}+210\,Tp^2\,Th^4\,Tv-42\,Th^2\,Tp^5-14\,Tp^6\,Th-70\,Tp^4\,Th^3-70\,Th^4\,Tp^3-840\,Tp^3\,Th^2\,Tv^2\\-42\,Th^5\,Tp^2-14\,Th\,Tv^6+84\,Tv^5\,Tp\,Th+280\,Tp^3\,Th^3\,Tv-420\,Tp\,Th^2\,Tv^4-672\,Tv^2\,Th^5\\+Tv^7-Tp^7+128\,Th^7-210\,Tp^2\,Th\,Tv^4+84\,Tv\,Tp^5\,Th+280\,Tp^3\,Th\,Tv^3+420\,Tp^2\,Th^2\,Tv^3+210\,Tp^4\,Th^2\,Tv\\+1344\,Tp\,Th^5\,Tv-210\,Tp^4\,Th\,Tv^2+1120\,Tp\,Th^3\,Tv^3-1680\,Tp\,Th^4\,Tv^2-1680\,Tp^2\,Th^3\,Tv^2+448\,Tv\,Th^6\\+84\,Th^2\,Tv^5-280\,Th^3\,Tv^4-35\,Tv^4\,Tp^3-21\,Tv^2\,Tp^5+21\,Tv^5\,Tp^2+35\,Tv^3\,Tp^4+560\,Th^4\,Tv^3-448\,Tp\,Th^6\end{gathered}}{1260\,Tv^2\,Th^2\,Tp^2}$$

$$E(TB)6 = -\frac{-105\,Tv\,Th^2 - 15\,Tv\,Tp^2 - 30\,Tv^3 + 45\,Th^3 + 35\,Tp\,Th^2 + 3\,Tp^3 + 15\,Tp^2\,Th - 60\,Tv\,Tp\,Th}{90\,Tv^2}$$

So these are the possible values for E(TB) according to the first case (case (1)). The resulting equations may a bit seems long, but they are really easy to calculate by a machine even with hand. For the remaining cases the mathematical process of the continuous approximation and calculation is the same. Because of the limitation of the paper pages we can not present here the remaining results.
But no matter, the most important thing in our paper is to show how we can model the travel time using a purely continuous approximation which is really easier to handle and estimate than other discrete models. Also results of this approach are very close to those obtained by discrete models but in really less time for calculation. Next we show how it is possible to generalize this approach to other cycle times.

7 Generalization of the continuous approximation :

One Depth AS/RSs can have various physical configurations such as Unit Load AS/RS (UL-AS/RS), Mobile Rack AS/RS (MR-AS/RS), Multi Aisles AS/RS (MA-AS/RS) but all have a common way of functioning for the S/R machine and its basic moves and travel times. Also, we have shown that a cycle time of the machine could be :

- Single Cycle (SC) and we know how to calculate it;
- Dual Cycle and we have presented its analytical modeling in section 6, and showed that it is basically composed of a Single Cycle E(SC) plus a Time Between E(TB). E(DC)=E(SC)+E(TB);
- Multi Cycles.

Now we know that a Dual Cycle is composed of a Single Cycle plus a Time Between, if we suppose that the S/R machine has a multi-loads capacity noted *m* which will allow it to handle *m* items and to do Multi Cycles, we'll have the following functioning : the machine starts from P/D station, goes to store up to *m* items in random bins and retrieve up to *m* items from random bins and finally come back to P/D station. That way we can easily notice that there is basically a Single Cycle and so an E(SC) plus several Time Between exactly *m*-1 and so (*m*-1)*E(TB), in the same way as in the Dual Cycle mode at the difference that in DC mode the machine will make only one Time Between. Thus, we can generalize our approach to model the cycle time in Multi Command mode noted E(MC) by a continuous approximation for One Depth AS/RSs as following :

$$E(MC)=E(SC)+E(TB)*(m-1)$$

Conclusion :

Storage is one of the most important activities in Flexible Manufacturing Systems (FMS). These FMS aim to maximally reduce the costs of the storage activity mainly in terms of time and space. Due to that, a good and well adapted conception of the AS/RS is required to reach this goal. This suppose the choice of the right configuration and appropriate equipments. The S/R machine is the main mean because of its travel time which is an important parameter that affects the whole system performances. For this reason we worked on this paper to model the cycle time of this machine using a purely continuous approximation. We started by presenting the possible functioning modes of the S/R machine (Single, Dual and Multi Command), its basic moves (horizontal on the main path, horizontal along the length of a rack, vertical along the height of a rack), the corresponding travel times (*Tp*, *Th*, *Tv*). We presented after, the basic cycle time and showed that the expected mean travel time of the S/R machine in a Single Cycle E(SC) is the maximum between the horizontal travel time and the vertical travel time. We showed after that a Dual Cycle is the sum of a Single Cycle plus a Time Between defined as the time taken to move between two random bins. We applied our approach and showed how to calculate the expected mean Time Between with a continuous approximation by considering the surface of a rack as a continuous plan and by approximating random discrete variables having uniform discrete distribution by continuous variables having a continuous uniform distribution. Our modeling of this E(TB) is valid for any random bins. By the same reasoning we showed that a Dual Cycle could be a Multi Cycle in case where the S/R machine would have a multi-loads capacity. Indeed, a Multi Cycle equals to do a Single Cycle plus several Time Between. If the machine has an m capacity it will make m-1 Times Between and consequently the expected mean travel time in a Multi Cycle E(MC) is the sum of expected mean travel time in a Single Cycle E(SC) and expected mean Time Between E(TB)*m-1.

Our approach in this paper is considered to be a general approach to model and calculate the travel time of the S/R machine by calculating the Time Between in order to use it for the estimation of Dual Cycle time and to easily generalize it for the calculation of Multi Cycles time, and finally to generalize this approach to the different One Depth AS/RS physical configurations. By this work we aim to :

1) Show that previous models (SARI [1] + HWANG [7]) are exact models but heavy for calculation.

2) Our modeling is approximate but : correct, very close to the previous exact models, really easy to calculate and particularly scientifically (mathematically) based approach.

3) Our method is easily generalizable to different One Depth AS/RS configurations.

4) By this easy generalization we aim to make our approach an easy to use/easy to calculate standard approach for the calculation and modeling of cycles time for One Depth AS/RS.

5) Using this standard we aim to study the optimization of different AS/RS dimensioning and by the way use the results for future conception of AS/RS.

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