THE STORAGE REPLENISHMENT PROBLEM IN RECTANGULAR WAREHOUSES

Haldun Süral, Ph.D.
Department of Industrial Engineering, Middle East Technical University, 06800 Ankara, Turkey

Claudia Archetti, Ph.D.
Department of Economics and Management, Universitá degli Studi di Brescia, 25122 Brescia, Italy

Melih Çelik, Ph.D.
Department of Industrial Engineering, Middle East Technical University, 06800 Ankara, Turkey

Abstract

In warehouses, storage replenishment operations involve the transportation of items to capacitated item slots in forward storage area from reserve storage. These items are later picked from these slots as their demand arises. While order picking constitutes the majority of warehouse operating costs, replenishment operations might be as costly in warehouses where pick lists generally consist of only a few lines (e.g., order fulfillment warehouses).

In this study, we consider the storage replenishment problem in a parallel-aisle warehouse, where replenishment and order picking operations are carried out in successive waves with time limits. The aim is to determine the item slots that will be replenished and the route of the replenishment worker in each replenishment wave, so as to minimize the total labor and travel costs, and ensure the availability of items at the start of the wave they will be picked. The problem is analogous to the inventory routing problem due to the inherent trade-off between labor and travel costs.

We present complexity results on different variants of the problem and show that the problem is NP-hard in general. Consequently, we use a heuristic approach inspired by those from the inventory routing literature. We use randomly generated warehouse instances to analyze the effect of different storage policies (random and turnover-based) and demand patterns (highly skewed or uniform) on replenishment performance, and to compare the proposed replenishment approach to those in practice.

1 Introduction

Breakdown of warehouse operating costs reveals that making necessary items available in pick area and picking of these items for satisfying customer orders cover 55% of the total warehousing costs. Although the order picking problem (OPP) is a well-studied problem, to the best of our knowledge, its relation with replenishment activities is not considered in the warehousing literature. In this study, we consider a coordinated approach, where the replenishment and transportation/routing decisions are made in an integrated manner by taking into account the dependence between the replenishment and pick cycles. In doing so, we aim to complete the replenishment operations within pre-specified time limits before picking operations, hence avoiding overtime, while minimizing the transportation costs by making use of economies of scale during replenishment.

Order picking operations inherently require items to be available in the storage area prior to picking. Upon receipt, items are put away into the reserve storage area, where they are stored in bulk amounts. The motivation for such a storage area is to preserve space efficiency at the cost of less efficient picks. Because of this, order picking is not performed in this area. Upon need, which is based on the picking schedule, items are broken down and replenished into the forward storage area, where storage is in smaller quantities. Here, the motivation is to sacrifice space efficiency and provide better accessibility of items, which leads to more efficient picking. Hence, the availability of items in the forward storage area is ensured by replenishing the needed items from the storage area.

In general, replenishment and picking activities are performed in sequence in a cyclic manner, particularly, in warehouses that employ manual order picking. Each of the replenishment and pick cycles is called a "wave." In practice, when planning for replenishment, these waves are treated independently, that is, the decisions of which items to replenish and how much are made based only on the upcoming pick wave. In this case, routing decisions for replenishment in the forward storage area are made mostly identical to those of order picking. However, in this case, two issues might arise: (i) The replenishment wave might exceed the time limit, resulting in the need for overtime, and (ii) Treating each wave independently might result in excessive transportation.

There are many studies in the literature that consider the combined effect of the building blocks of warehouse design and management. For instance, Thomas and Meller (2014) emphasize the benefit of the integration of put-away, replenishment, and order picking in the design of warehouses. Strack and Pochet (2010) present an integrated model for the warehouse and inventory management problems at the tactical level. The model decides on the replenishment amounts of items in addition to deciding which items are assigned to the reserve storage and forward areas, as well as which items are picked from which area, etc. To the best of our knowledge, the case of replenishment of items in the forward storage area is not considered in conjunction with order picking cycles, which is a gap that is aimed to be filled in this study.

2 Problem Definition and Complexity

A typical schedule for t days in a warehouse resembles the one in Figure 1, where the replenishment and picking activities are performed in a cyclic manner. For simplicity, we assume equal cycle lengths and the customer orders to be picked in each pick cycle are known for t days.

		Before 08:00	08:00-10:00	10:00-12:00	12:00-14:00	14:00-16:00	16:00-18:00	18:00-20:00
Day 1	→	Items received into the reserve storage area	Replenishment	Picking	Replenishment	Picking	Replenishment	Picking
Day 2	→	Items received into the reserve storage area	Replenishment	Picking	Replenishment	Picking	Replenishment	Picking
...	...	...	...	...	...	...	...	...
Day *t*	→	Items received into the reserve storage area	Replenishment	Picking	Replenishment	Picking	Replenishment	Picking

Figure 1: A typical schedule for *t* days of replenishment and order picking waves

We define the *storage replenishment problem* (SRP), which aims to make decisions on (i) when and how much to replenish each item in the pick area from reserve storage to guarantee availability and (ii) the routing of the replenishment carrier in each period. In the SRP, we assume a single uncapacitated replenishment carrier available. A set of "periods" (waves), their pre-specified time lengths of waves, and arrivals of each item at the reserve storage in each day are known. The warehouse layout, item locations with corresponding storage capacities, initial inventories of each item in the reserve and forward storage areas, and the amount of each item to be picked in each wave are given. Demand patterns (uniform or skewed) and storage policies (random or turnover-based) are specified and known. The objective in the SRP is to minimize the total replenishment travel time. Under these settings, the SRP is similar to the Inventory Routing Problem (IRP). The (dis)similar features that (de)construct the two problems can be specified as follows. There exists one-to-one correspondence between "supliers" and "retailers" in the IRP and "reserve storage area" and "item locations". "Demand time and amounts" in the IRP match with "pick lists" of the SRP while "load capacity" of IRP can represent "wave time limit" of the SRP. A "single item and multiple retailers" structure of the IRP can be seen as a "multiple items, each of which demanded by one retailer". Although "holding cost" of the IRP is an important trade-off component, the SRP does not incorporate such a cost item. However, "availability" may point out an imputed cost in the SRP. Another distinction between two problems is about routing decisions because the SPR has a special structure that makes routing "easir" than that of the IRP. Below we introduce a few variates of the SPR and discuss their computational complexity status.

Theorem 1 *If feasible, the SRP is polynomially solvable for a single wave.*

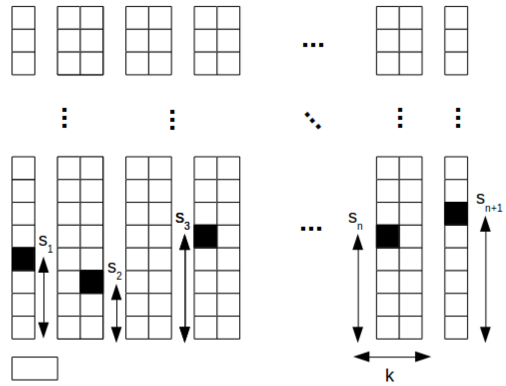

Figure 2: The SRP instance required for transformation from the PARTITION problem

Proof. Without the existence of multiple periods, the SRP is equivalent to the OPP, which is polynomially solvable by Ratliff and Rosenthal (1983). □

Theorem 2 *The SRP is weakly $\mathcal{NP}$-hard for two periods.*

Proof. $\mathcal{NP}$-hardness by transformation from PARTITION (a weakly $\mathcal{NP}$-hard problem), which, given a set A and an integer size s_i for each $a \in A$, seeks a subset $A' \subseteq A$ such that the total size in A' equals the total size in $A \setminus A'$.

The required transformation uses the SRP instance in Figure 2, and sets the following parameters:

- One item in each of the $n + 1$ aisles
- All item locations have 1 unit capacity
- Items $1, 2, \ldots, n$ to be picked in the second wave
- Item $n + 1$ to be picked in both waves
- Wave time limit: $2(s_{n+1} + k) + \sum_{i=1}^{n} s_i$

It is easy to observe that we can find a feasible solution to the SRP instance only if the corresponding PARTITION instance with item "sizes" s_i for $i = 1, 2, \ldots, n$ and $s_{n+1} + k$ for $i = n + 1$ has a feasible solution. This shows that unless $\mathcal{P} = \mathcal{NP}$, the problem can be solved in pseudo-polynomial time in the best case. To solve the problem in pseudo-polynomial time, one needs to extend the state definition by Ratliff and Rosenthal (1983) to include remaining time for both tours in the two replenishment waves. Here, the number of states (which is constant for the Ratliff and Rosenthal algorithm) depends on the number of pick aisles and the wave time limit. Consequently, it is polynomial in the case of unary

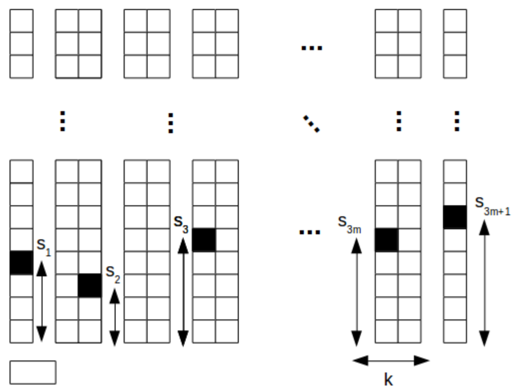

Figure 3: The SRP instance required for transformation from the 3-PARTITION problem

representation of the inputs, which implies that it can be solved in pseudo-polynomial time. □

This yields an important corollary for the OPP with two capacitated order pickers.

Corollary 3 *The OPP with two capacitated pickers is weakly $\mathcal{NP}$-hard.*

Next, we show the strong $\mathcal{NP}$-hardness of the SRP with at least three periods.

Theorem 4 *The SRP is strongly $\mathcal{NP}$-hard for more than two periods.*

Proof. $\mathcal{NP}$-hardness by transformation from 3-PARTITION, where we have set of $3m$ elements, an integer bound B, a size s_i for all items in the set such that $\frac{B}{4} < s_i < \frac{B}{2}$ holds for all i, and all sizes in the set add up to mB. We are looking for a partition of the set into m disjoint sets such that all sets have a total size of exactly B units each.

The required SRP instance for transformation is given in Figure 3.

The following settings transform the SRP instance in the figure to an equivalent 3-PARTITION instance:

- One item in each of the $3m + 1$ aisles
- All item locations have 1 unit capacity
- Items $1, 2, \ldots, 3m$ to be picked in the last wave
- Item $3m + 1$ to be picked in all waves
- $\frac{B}{8} < s_i < \frac{B}{4}$ for all i and $\sum_{i=1}^{3m} s_i = \frac{B}{2}$
- Wave time limit: $2((3m + 1)k + s_{3m+1}) + B$

Here, a feasible SRP solution can be found only if the 3-PARTITION instance with item weights equal to $i = 1, 2, \ldots, 3m$ and $s_{3m+1} + k$ for $i = 3m + 1$ has a feasible solution. Since the 3-PARTITION problem is strongly $\mathcal{NP}$-hard, so is the SRP with at least three periods. $\square$

Theorem 4 results in an important corollary for the OPP with at least three capacitated pickers.

Corollary 5 *The OPP with more than two capacitated pickers is strongly $\mathcal{NP}$-hard.*

3 An A Priori Route-Based Heuristic

The NP-completeness of the SRP suggests that as the instance size increases, the computational burden to solve it to optimality will substantially increase. To overcome this burden, we propose an a priori route-based heuristic in this section. The motivation behind this heuristic is that once the items to visit are fixed, the routing problem is easily solvable.

In the first step of the heuristic, we solve the OPP corresponding to all the items that will be picked throughout the planning horizon. To do so, we may use the exact approach by Ratliff and Rosenthal (1983), or one of the heuristics proposed by Hall (1993). The possibility of solving the a priori routing problem in a fast manner differentiates our work from that by Solyalı and Süral (2011), where the routing subproblem requires the solution of the TSP on a general graph using a specialized solver (Concorde, 2011).

Once the a priori route is determined, we fix the sequence of items to be visited in each replenishment trip. For each item $i \in M$, we define α_i and β_i, which consist of the items and the P&D point that precede and succeed the item on the a priori route, respectively. By fixing the sequences of item replenishment, the main aim is to simplify the routing decisions in the next step.

An example for using the a priori route for determining the replenishment sequence is given in Figure 4, where an instance with 16 items on a warehouse with 8 aisles is considered. The figure on the left shows the optimal tour (found using the Ratliff and Rosenthal algorithm) that visits all 16 items. If the next replenishment wave involves the 7 items shown in blue, we use the sequence on the optimal your to determine the sequence at which these items will be replenished. The resulting tour is shown on the right.

The first two steps of the heuristic determine the route of replenishment, given which items will be replenished. This leaves the decision of which items to replenish (note that due to the order-up-to policy, how much to replenish is not a part of the decisions). For this end, we extend the strong formulation of the reduced model for the IRP, for which the idea was first put forward by Pınar and Süral (2006) and later applied by Solyalı and Süral (2011).

The notation we use for the restricted model is given in Table 1. The index set M represents the items (stock locations), whereas M' additionally includes the P&D point.

Table 1: Notation

Index sets

M	Stock locations (items)
M'	$M \cup v_0$
T	Waves
T'	$T \cup \{\lvert T\rvert + 1\}$
T^*	$T \cup \{0\}$
α_i	Predecessors of $i \in M'$ on the a priori route
β_i	Successors of $i \in M'$ on the a priori route

Parameters

c_{ij}	Travel time between $i \in M'$ and $j \in M'$
p_{it}	Arrivals of item $i \in M$ at the reserve storage at wave $t \in T$
b_{ikt}	Amount to be sent to stock location $i \in M$ in wave $t \in T'$ if the last replenishment was made in wave $k \in T^*$
r_{it}	Amount of item $i \in M$ to be picked in wave $t \in T$
π_{it}	The earliest wave for item $i \in M$ in which a replenishment can satisfy the demand from wave π_{it} to wave $t \in T$
μ_{ik}	The latest wave for item $i \in M$ for which a replenishment in wave $k \in T$ can satisfy the demand
C	Total time allowed for each wave
I_{i1}	Initial inventory of stock location $i \in M$
I'_{i1}	Initial inventory of item $i \in M$ in the reserve storage area

Decision variables

z_{it}	Binary variable indicating whether stock location or P&D point $i \in M'$ is visited in wave $t \in T$
$\hat{y}^t_{ij}$	Binary variable indicating whether $j \in M'$ follows $i \in M'$ in the route for wave $t \in T$
I_{it}	Inventory of item $i \in M$ in forward storage in wave $t \in T'$
I'_{it}	Inventory of item $i \in M$ in reserve storage in wave $t \in T'$
w_{ikt}	Binary variable indicating whether stock location $i \in M$ is replenished in wave $t \in T'$, given that the last replenishment was made in wave k, $\pi_{it} \leq k \leq t - 1$.

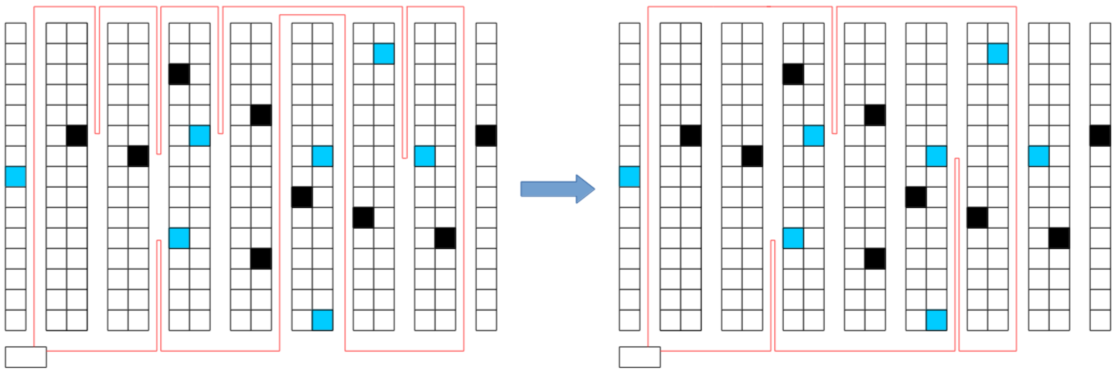

Figure 4: (a) The optimal route involving all 16 items, (b) The resulting route when a subset of items (shown in blue) are to be replenished

Whereas use T to denote the waves, the sets T' and T^* also include $\{0\}$ and $|T| + 1$, respectively. The predecessor and successor sets for stock location or P&D point $i \in M'$ are denoted by α_i and β_i, respectively.

Travel time between $i \in M'$ and $j \in M'$ is given by c_{ij}. If $i \in M$, this may also include the time to replenish the item. At the beginning of each day (corresponding to wave $t \in T$), p_{it} units of item $i \in M$ arrive at the reserve storage area. In each wave $t \in T$, r_{it} units of item $i \in M$ are to be picked. The initial inventory of item $i \in M$ at the reserve and forward storage areas are given by I'_{i1} and I_{i1}, respectively.

To incorporate the strong formulation, we make use of additional parameters. The parameter b_{ikt} denotes the amount to be sent to stock location $i \in M$ in wave $t \in T'$ if the last replenishment was made in wave $k \in T^*$. More formally,

$$b_{i0t} = U_i - I_{i1} + \sum_{j=1}^{t-1} r_{ij}, \text{ and}$$

$$b_{ikt} = \sum_{j=k}^{t-1} r_{ij} \text{ for all } k \in T.$$

For $k = 0$, the amount b_{ikt} should increase the initial inventory level I_{i1} for item $i \in M$ to the order-up-to level U_i and needs to satisfy the demand for the first $t-1$ waves. For $k \geq 1$, it needs to satisfy the demand between waves k and $t-1$.

Since we need to guarantee that none of the storage locations stocks out during the planning horizon, two additional parameters are used: (i) π_{it} is the earliest wave for item $i \in M$ in which a replenishment in wave $k \in T^*$ can satisfy the demand of periods $k+1, k+2, \ldots, t-1$ until the next replenishment in wave $t \in T$, (ii) μ_{kt} is the latest wave for item $i \in M$ for which a replenishment in wave $k \in T$ can satisfy the demand in waves

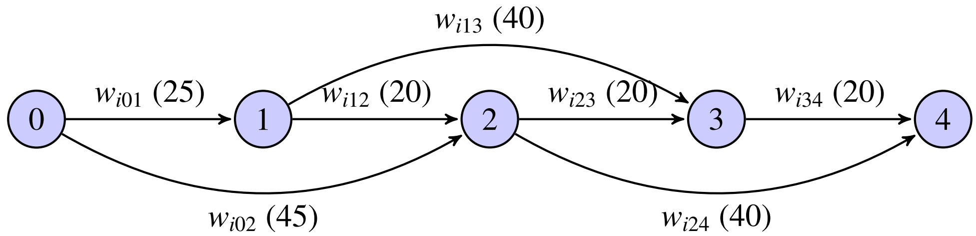

Figure 5: The shortest path network corresponding to the given example, with the b_{ikt} values given in brackets

$k+1, k+2, \ldots, t-1$ until the next replenishment in wave $t \in T$. In mathematical terms:

$$\pi_{it} = \max_{0 \leq k \leq t-1} \{k : \ b_{ikt} \leq U_i\}, \text{ and}$$
$$\mu_{kt} = \max_{k+1 \leq t \leq |T|+1} \{t : \ b_{ikt} \leq U_i\}.$$

We use decision variable w_{ikt} to denote whether item $i \in M$ is replenished in wave $t \in T'$, after the last replenishment was made in wave k, where $\pi_{it} \leq k \leq t-1$. Here, w_{i0t} denotes whether the first replenishment for item i is in wave t and $w_{ik,|T|+1}$ indicates whether the last replenishment for item i is in wave k.

The replenishment scheme for item $i \in M$, when modeled with the binary decision variables w_{ikt}, constitutes a shortest path network. As an example, consider an item with an initial inventory of 20 units, stock location capacity of 45 units, and assume that a constant amount of 20 units is picked in every wave. For a horizon of 3 waves, this yields the network given in Figure 5.

The decision variable z_{it} indicates whether item location $i \in M$ or the P&D point is visited in wave $t \in T$, $\hat{y}^t_{ij}$ is the binary routing variable for wave $t \in T$, and the variables I_{it} and I'_{it} denote the inventory of item $i \in M$ in the forward and reserve storage areas, respectively.

Using the given index sets, parameters, and decision variables, we solve the following model to find the set of items to replenish in each wave:

$$min \quad \sum_{i \in M'} \sum_{j \in M'} \sum_{t \in T} c_{ij} \hat{y}^t_{ij} \tag{1}$$

$$s.t. \quad I'_{i,t+1} = I'_{it} + p_{it} - \sum_{k=\pi_{it}}^{t-1} b_{ikt} w_{ikt} \quad \forall i \in M,\ t \in T, \tag{2}$$

$$I'_{it} \geq \sum_{k=\pi_{it}}^{t-1} b_{ikt} w_{ikt} \qquad \forall i \in M,\ t \in T, \tag{3}$$

$$I_{i,t+1} = I_{it} + \sum_{k=\pi_{it}}^{t-1} b_{ikt} w_{ikt} - r_{it} \quad \forall i \in M,\ t \in T, \tag{4}$$

$$\sum_{k=1}^{\mu_{i,0}} w_{i0k} = 1 \qquad \forall i \in M, \tag{5}$$

$$\sum_{k=t+1}^{\mu_{it}} w_{itk} - \sum_{k=\pi_{it}}^{t-1} w_{ikt} = 0 \qquad \forall i \in M,\ t \in T, \tag{6}$$

$$\sum_{k=\pi_{it}}^{|T|} w_{i,k,|T|+1} = 1 \qquad \forall i \in M, \tag{7}$$

$$\sum_{k=\pi_{it}}^{t-1} w_{ikt} = z_{it} \qquad \forall i \in M,\ t \in T, \tag{8}$$

$$z_{it} \leq z_{0t} \qquad \forall i \in M,\ t \in T. \tag{9}$$

$$\sum_{j \in \alpha_i} \hat{y}^t_{ij} = z_{it} \qquad \forall i \in M',\ t \in T, \tag{10}$$

$$\sum_{j \in \beta_i} \hat{y}^t_{ji} = z_{it} \qquad \forall i \in M',\ t \in T, \tag{11}$$

$$\sum_{i \in M'} \sum_{j \in M'} c_{ij} \hat{y}^t_{ij} \leq C \qquad \forall t \in T, \tag{12}$$

$$w_{ikt} \in \{0, 1\} \qquad \forall i \in M,\ t \in T', \pi_{it} \leq k \leq t-1, \tag{13}$$

$$\hat{y}^t_{ij} \in \{0, 1\} \qquad \forall i \in M',\ j \in M', t \in T, \tag{14}$$

$$z_{it} \in \{0, 1\} \qquad \forall i \in M',\ t \in T, \tag{15}$$

$$I_{it},\ I'_{it} \geq 0, \qquad \forall i \in M,\ t \in T. \tag{16}$$

Here, objective function (1) minimizes the total replenishment time. Constraints (2) and (3) impose inventory balance at the reserve storage area, whereas Constraint (4) stipulates inventory balance at the stock locations. Constraints (5)-(7) are the flow balance constraints for the resulting shortest path network, whereas Constraint (8) ensures that an outgoing arc in the shortest path network results in the corresponding binary replenishment variable to be 1. Using Constraint (9), we impose the inclusion of the P&D point in each replenishment tour, while Constraints (10) and (11) connect the routing and binary replenishment variables, and impose the tour precedence relations. Wave time limits are set by Constraint (12), and Constraints (13) through (16) indicate the ranges for the decision variables.

The modified model uses the precedence sets for each item determined by the previous step and the w_{ikt} values for each item as additional parameters, and decides on whether an arc on the network will be used or not, subject to inventory balance, network flow balance, wave time limit, and routing precedence constraints. As the last step, given which items will be replenished in each wave, the resulting routes are improved using the Ratliff and Rosenthal algorithm for these waves.

4 Computational Experiments

The objectives of our computational experiments in this study can be summarized as follows: (i) compare different a priori routing approaches (optimal, S-shape, largest gap) among each other; (ii) measure the effect of instance size, demand structure, and storage policies on the efficiency of replenishment; and (iii) analyze the extent of the improvement of the proposed replenishment schemes over those in practice. Our instances are based on those by Archetti et al. (2007) and Solyalı and Süral (2011). The settings are as follows. Warehouse setting assumes (i) 10 aisles and 15 item locations in each aisle and (ii) successive locations (aisles) are 1(2.5) unit(s) of travel time apart. Number of periods (H) are 3 or 4 where they are "3 days with 1 wave each", "1 day with 3 waves", "4 days with 1 wave each", "2 days with 2 waves each", and "1 day with 4 waves". Number of items (n) are 15, 30, or 75. Per-period demand (r_i) is set as $\sim$ $Disc\,Unif(10, 100)$. Initial inventory at reserve storage is set as $\sim$ $Disc\,Unif(0.8r_i, 1.2r_i)$ while order-up-to levels (U_i) are put as $g_i r_i$, $g_i \in \{2, 3\}$. Daily item arrivals are set as $\sim$ $Disc\,Unif(0.8U_i, 1.2U_i)$ while initial inventory at item location equals to $U_i - r_i$. We set wave time limit as 180 time units and solve 5 test instances for each setting. In total we solve a set of 75 instances.

We consider two types of demand: (1) uniform, where the probability that any item will appear on a pick list on a given day is identical; and (2) skewed, where certain items have more likelihood to appear on the pick list. For the latter, we use the Pareto distribution with 20-80 skewness, which implies that 20% of the items in the warehouse receive 80% of the total demand. In this case, we analyze two storage policies: (i) random storage, where items are stored in the warehouse randomly, regardless of the demand; and (ii) turnover-based storage, where items that generate more demand are stored "in convenient locations."

Our instances have been run on a computer with Intel Core i7-4500U CPU at quad-core 1.80GHz and 8 GB RAM. All CPU times are within 5 seconds.

We first consider the effect of a priori route. Figure 6 shows the percent deviations of heuristic a priori route schemes from the optimal a priori route solutions. Here, regardless of the number of periods, we observe opposite patterns for the S-shape and largest gap heuristics. The S-shape heuristic is closer to the optimal route when the density of the items in the warehouse is high. This is reflected in the results by the decreasing gap of this heuristic from 15 to 75 items. The largest gap heuristic, which performs better when item density is lower, displays an increasing gap level with increasing number of items.

Now we consider the effect of demand and storage policies. The best results are obtained when demand is skewed and turnover-based storage is used. Figure 7 shows the percent deviations of the other demand patterns and storage policies from this case. There are two conclusions which can be drawn from the figure: (1) if demand is skewed, applying a turnover-based storage has an average travel time savings of 6% on average, underlining the importance of using a storage policy in line with demand skewness. (2) when demand is skewed and turnover-based storage is applied, an average of 4% less travel time is observed compared to uniform demand and random storage. Hence, when storage policy is

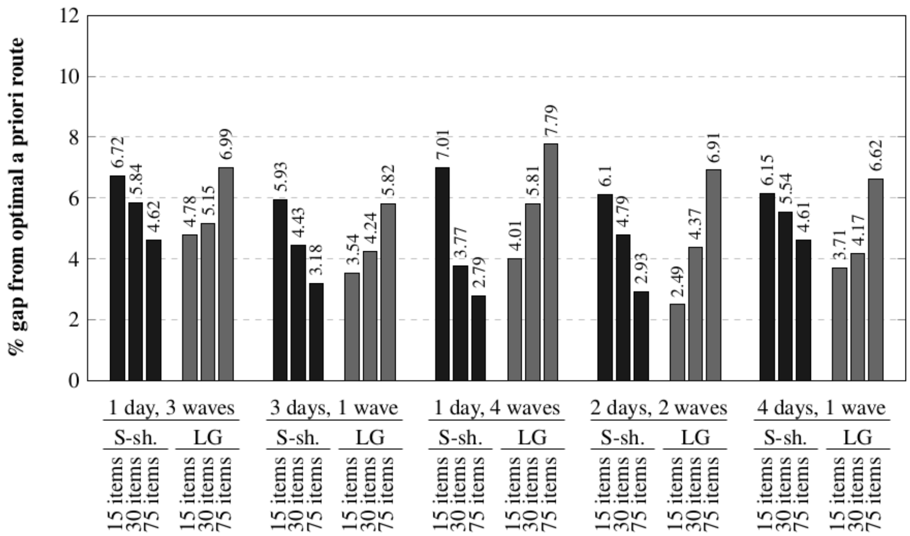

Figure 6: Deviations of heuristic a priori route schemes from the optimal a priori route solution

in line with demand skewness, skewed demand pattern yields a slight advantage over the case when demand is uniform.

In Table 2, we show the percent improvement of the proposed heuristics under different a priori route schemes from the method where each wave is treated independently as being treated in practice. As the table also indicates, the percent improvement ranges from 15% to 31% for optimal, from 9% to 28% for S-shape, and from 9% to 23% for largest gap a priori routes.

5 Conclusion and Further Research Directions

This study demonstrates that item replenishment in warehouses can be made more efficient by considering waves in a coordinated way. We define the storage replenishment problem (SRP), which aims to determine the replenishment sequences and amounts for each wave so that total travel time for replenishment is minimized. We establish the complexity results for this problem and show that proposed approaches outperform those in practice by more than 20% on average.

Immediate further research directions involve the extension of the work to multiple capacitated replenishment pickers and warehouses with different layouts (with middle aisles, fishbone aisles, etc.). Another interesting area to explore is the case where items that cannot be replenished from the reserve storage area being replenished from the receiving area at the expense of penalty costs, thereby resolving any infeasibility issues.

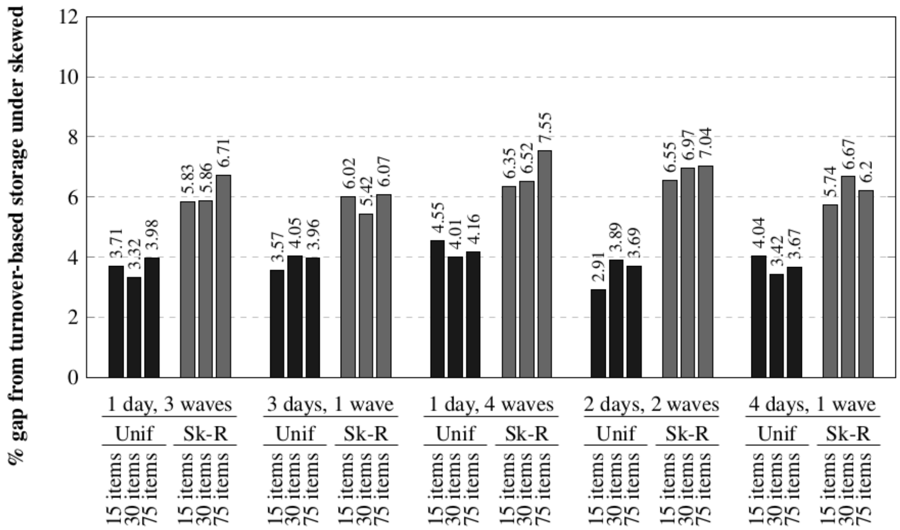

Figure 7: Deviations of the (i) uniform demand pattern (Unif) and (ii) skewed demand pattern with uniform storage policy from turnover-based storage under skewed demand

Table 2: Percent improvement of different a priori routing methods from practice

Periods	***n***	**Optimal**	**S-shape**	**Largest gap**
1 day, 3 waves	15	20.07	13.97	21.61
	30	24.04	17.64	18.66
	75	31.13	28.71	17.36
3 days, 1 wave	15	20.87	15.35	22.89
	30	26.64	19.30	19.26
	75	23.36	21.54	15.77
1 day, 4 waves	15	15.17	9.20	20.68
	30	23.81	16.90	17.12
	75	23.59	21.20	9.00
2 days, 2 waves,	15	15.79	11.51	18.54
	30	23.95	19.76	15.14
	75	23.00	20.99	9.03
4 days, 1 wave	15	17.09	10.69	23.62
	30	22.14	16.94	14.86
	75	30.71	27.69	12.49

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