IENG 4445 - Facilities Design
A Note on the Flow Dominance Measure

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The measure f of flow dominance is a number between 0 and 1 that indicates whether no dominant flows occur in a from-to matrix (the case where $f = 1$) or whether there are dominant flows (the case where $f = 0$).

If there are M processes, the from-to matrix is of order $M \times M$ and each entry is denoted by w_{ij} where $i = 1 \ldots M, j = 1 \ldots M$. There are M^2 entries in the from-to matrix. Recall that w_{ij} is a result of product volumes, routings, and equivalency factors.

The coefficient of variance of the matrix is:

$$\sqrt{\frac{\sum_{i=1}^{M}\sum_{j=1}^{M}(w_{ij}-\bar{w})^2}{(M^2-1)}}$$

Where:

$$\bar{w} = \frac{\sum_{i=1}^{M}\sum_{j=1}^{M} w_{ij}}{M^2}$$

The normalized coefficient of variance f' is given by:

$$f' = \frac{\sqrt{\frac{\sum_{i=1}^{M}\sum_{j=1}^{M}(w_{ij}-\bar{w})^2}{(M^2-1)}}}{\bar{w}}$$

However,

$$\sum_{i=1}^{M}\sum_{j=1}^{M}(w_{ij}-\bar{w})^2 = \sum_{i=1}^{M}\sum_{j=1}^{M}(w_{ij}^2+\bar{w}^2-2w_{ij}\bar{w})$$

$$
\begin{aligned}
&= \sum_{i=1}^{M}\sum_{j=1}^{M} w_{ij}^2 + M^2\bar{w}^2 - 2\sum_{i=1}^{M}\sum_{j=1}^{M} w_{ij}\bar{w} \\
&= \sum_{i=1}^{M}\sum_{j=1}^{M} w_{ij}^2 + M^2\bar{w}^2 - 2M^2\bar{w}^2 \\
&= \sum_{i=1}^{M}\sum_{j=1}^{M} w_{ij}^2 - M^2\bar{w}^2
\end{aligned}
$$

Thus,

$$
f' = \frac{\sqrt{\frac{\sum_{i=1}^{M}\sum_{j=1}^{M} w_{ij}^2 - M^2\bar{w}^2}{M^2-1}}}{\bar{w}} \tag{1}
$$

We can find the normalized coefficient of variance of two matrices, one with nearly all equal flows and another with a few dominant flows. For example, the following 4×4 matrix L has nearly all equal flows, and its normalized coefficient of variance, f_L is a lower bound on f'. Except for the diagonal elements which are 0, all other elements have a value of 1.

$$
L = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}
$$

An $M \times M$ matrix such as L has $\sum_{i=1}^{M}\sum_{j=1}^{M} w_{ij}^2 = M^2 - M$ and $\bar{w} = \frac{M^2-M}{M^2}$. Substituting these values in equation 1, we obtain:

$$
f_L = M\sqrt{\frac{1}{(M-1)(M^2-1)}} \tag{2}
$$

We now consider a matrix such as the 4×4 matrix U below which has a few dominant flows. Most of the matrix has a flow of zero. However, all elements of the first diagonal to the right of the main diagonal have flows of 1. If the from-to matrix resembles U, it is very easy to build a layout for the plant. The department flows are from $1 \rightarrow 2 \rightarrow 3 \rightarrow \ldots$ and therefore a linear or U-shaped layout might be very suitable.

$$
U = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

Note that an $M \times M$ matrix such as U has $\sum_{i=1}^{M}\sum_{j=1}^{M} w_{ij}^2 = M - 1$ and $\bar{w} = \frac{M-1}{M^2}$. Substituting these values in equation 1, we obtain:

$$f_U = M\sqrt{\frac{M^2 - M + 1}{(M^2 - 1)(M - 1)}} \tag{3}$$

For any from-to matrix, we now define its flow dominance measure f as follows:

$$f = \frac{f_U - f'}{f_U - f_L} \tag{4}$$

Clearly, f is a number between 0 and 1. If the matrix has highly dispersed flows such as in matrix L, $f' \rightarrow f_L$ and $f \rightarrow 1$. If the matrix has dominant flows such as in matrix U, $f' \rightarrow f_U$ and $f \rightarrow 0$

It is said that:

- If $f \rightarrow 0$, then a product layout is suitable.
- If $f \rightarrow 1$, then any layout is appropriate from a qualitative perspective which implies that qualitative factors should be investigated.
- if $0 \ll f \ll 1.0$, then either a process or group layout might be appropriate.