“Performance Analysis of Split-Case Sorting Systems” by Johnson and Meller

Main Points

- Split-case sorting system operation and technology.
- Use Bernoulli process to model induction process and characterize negative effect of inductor interference.
- Faster inductors should be placed upstream.
- Split induction systems can outperform side-by-side induction systems.
- Presorting can be used for increased throughput (but it can hurt picking).

“Performance Analysis of Split-Case Sorting Systems” by Johnson and Meller

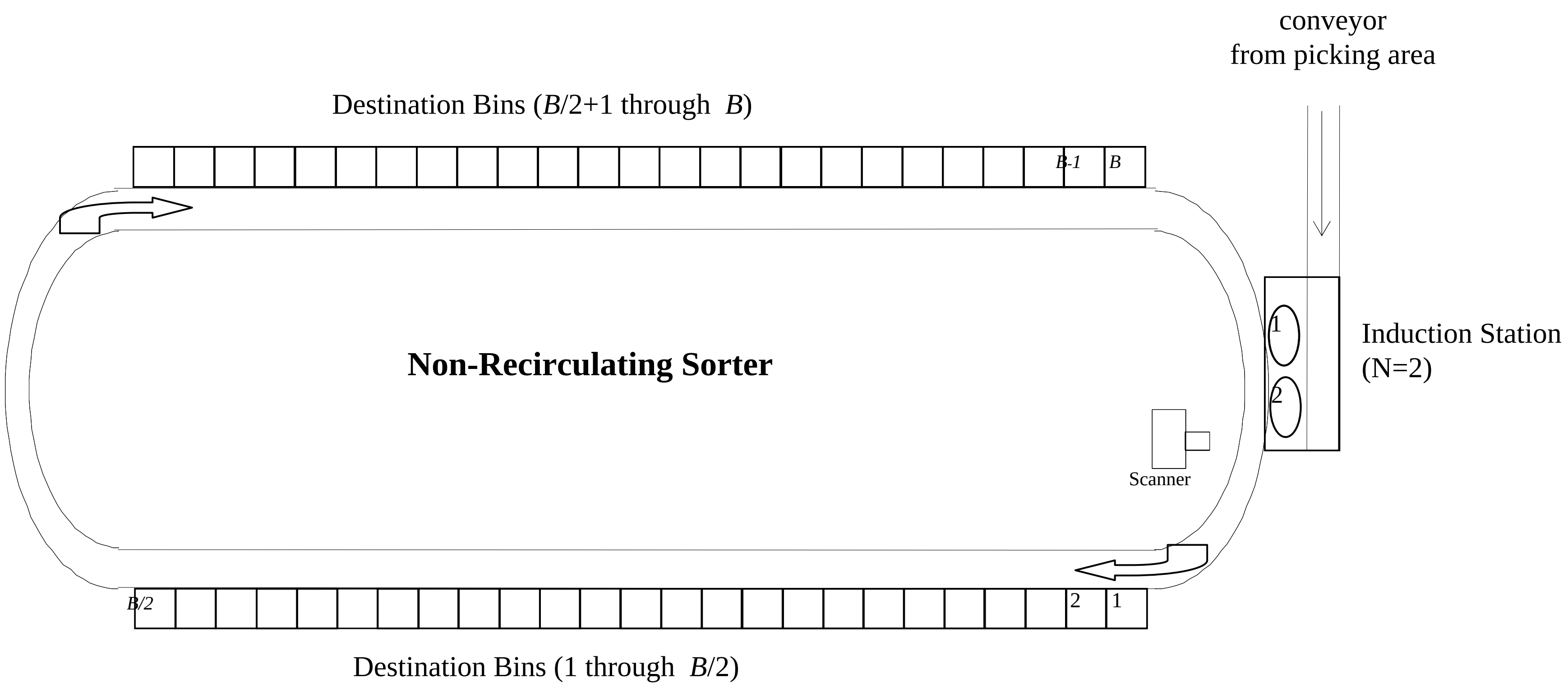
Introduction

- Used in order fulfillment centers (e.g., Amazon.com, L. L. Bean, or Sears).
- Need to fill lots of orders with common items.
- Can either go out and pick each order individually (stopping at the same location multiple times) or pick a batch of orders at a time and then sort them into individual orders.
- When order commonality is high enough, then batch picking of full cases and splitting the cases for sorting is efficient.

Source: W&H Systems, Inc.

Source: Cleco Systems, http://www.cleco.nl/objects/images/sortation.jpg

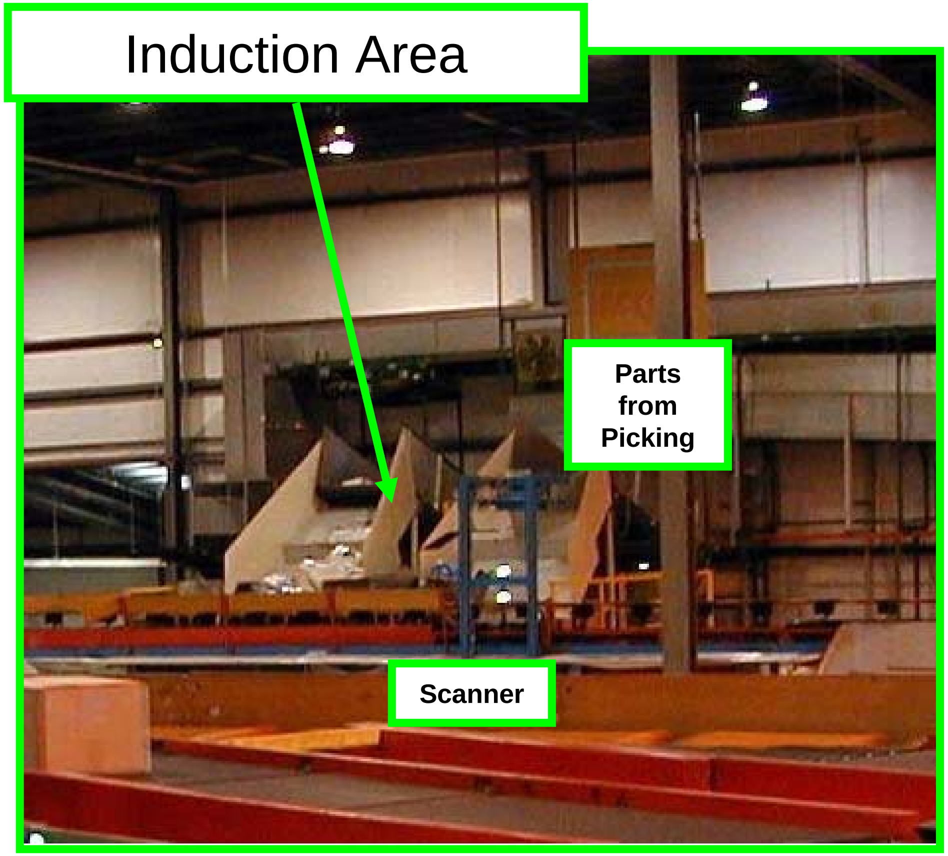
Automated Sorting System



Source: Johnson, M.E. and Meller, R. D., "Performance Analysis of Split-Case Sorting System," to appear in *M&SOM*.

Sorting System Sub-Systems

- Induction:
  - typically manual (humans)
  - can be automated (robots or conveyors)
- Sortation:
  - manual (humans) or
  - automated conveyors (tilt-tray, bomb-bay, cross-belt)
- Packing:
  - place items into shipping carton
  - check for all items (quality assurance)
  - add packing slip/invoice
  - button up



Source: Hanover Direct, Roanoke, VA

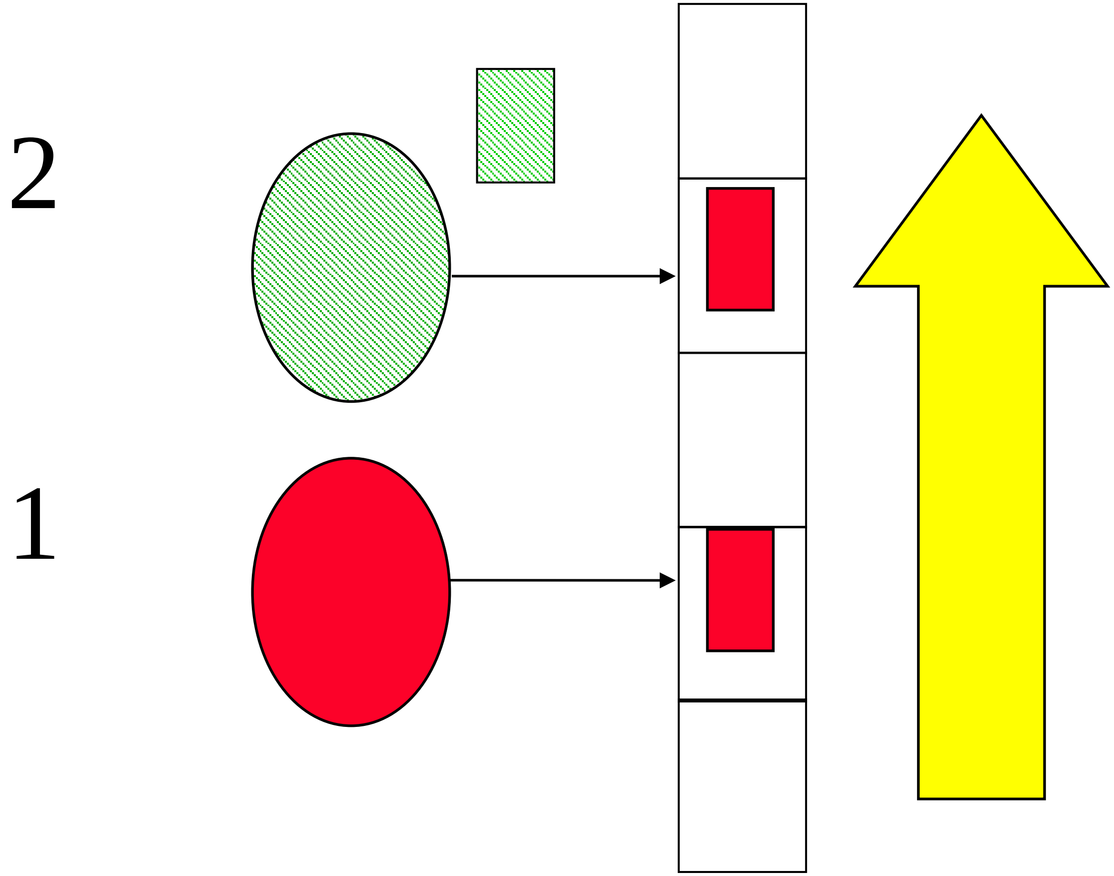
Induction Process

- Paper makes claim that induction process is the critical process/sub-system.
- Tends to limit throughput of the system once sorter hardware is in place.
- Decisions:
  - How many inductors?
  - Where to place them at stations and within station?
- Objective:
  - Would like to minimize cost (function of number of pickers and picking stations).
  - Need to meet throughput requirements.

Notation

- p_i = probability that inductor i can induct onto a moving conveyor ($i = 1, \ldots, N$)
- $p_i = \lambda_i / s, \lambda_i < s$
- λ_i = the induction rate of inductor i if working in isolation
- s = speed of the conveyor
- λ_i' = the effective induction rate of inductor i; $\lambda_i' \leq \lambda_i < s$

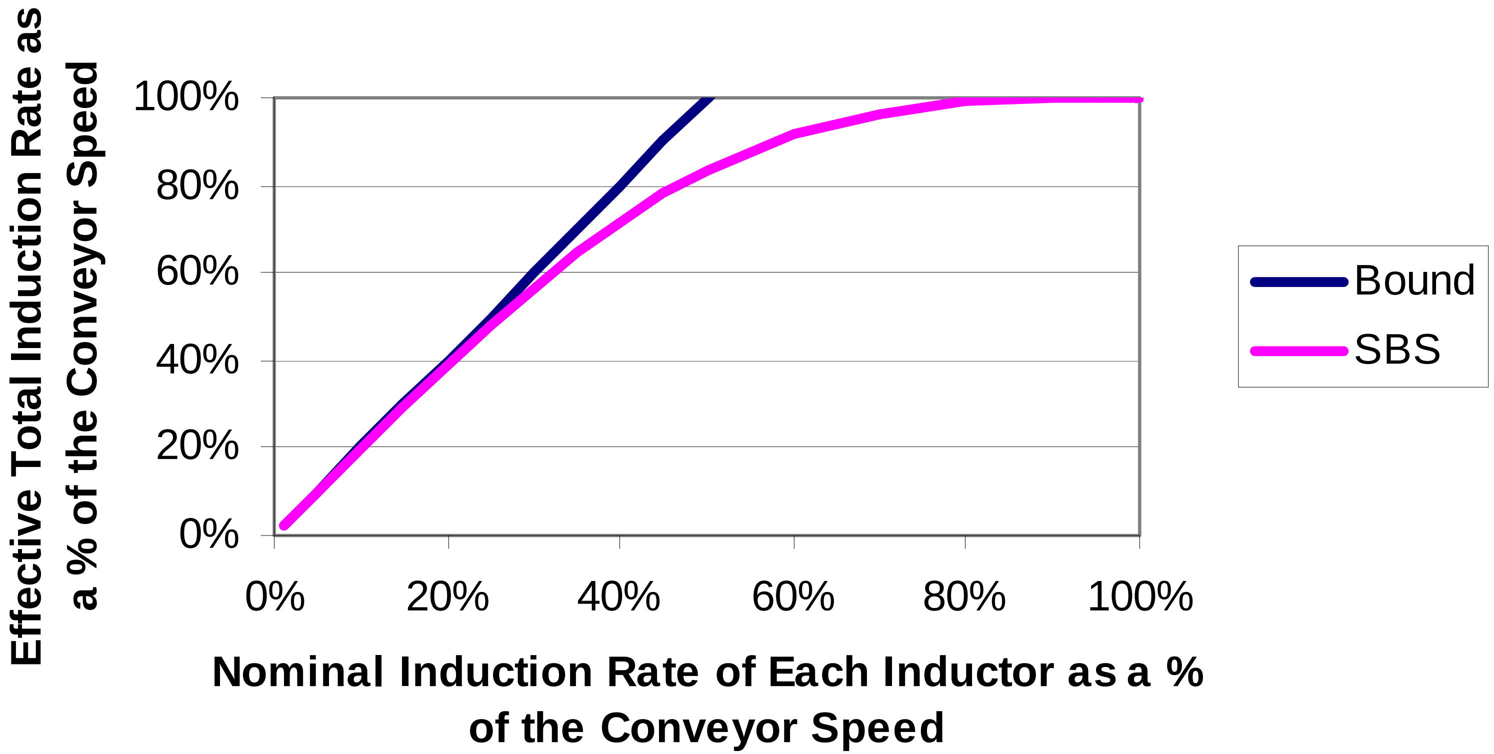
Side-by-Side Inductor Interference



Side-by-Side Inductors

- $\lambda_1' = \lambda_1$ since inductor 1 is never blocked
- What about inductor 2?
  - Geometric Distribution (p): Mean number of trials until first success equals $1/p$.
  - Mean number of trials until inductor 2 ready to place an item on $= 1/p_2 - 1$.
  - Mean number of trials until inductor 2 sees an empty tray $= 1/(1 - p_1)$.
  - Add these together and take the inverse ... yields the probability that inductor 2 hits the next tray.
  - Multiply by s and you have the effective induction rate of inductor 2: $\lambda_2' = [\frac{1}{\lambda_2/s} - 1 + \frac{1}{1-\lambda_1/s}]^{-1} s$.

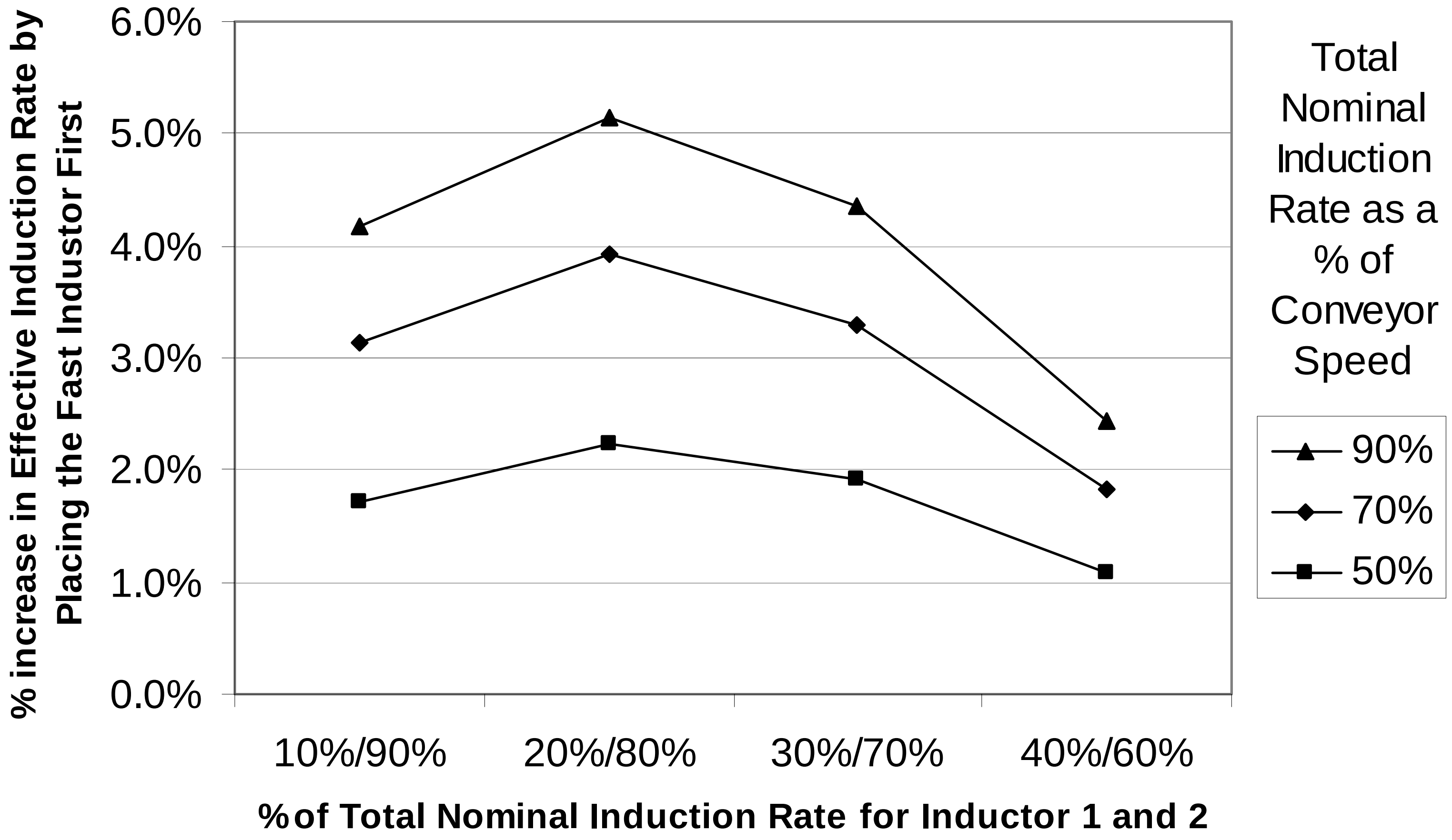
Side-by-Side Inductor Interference



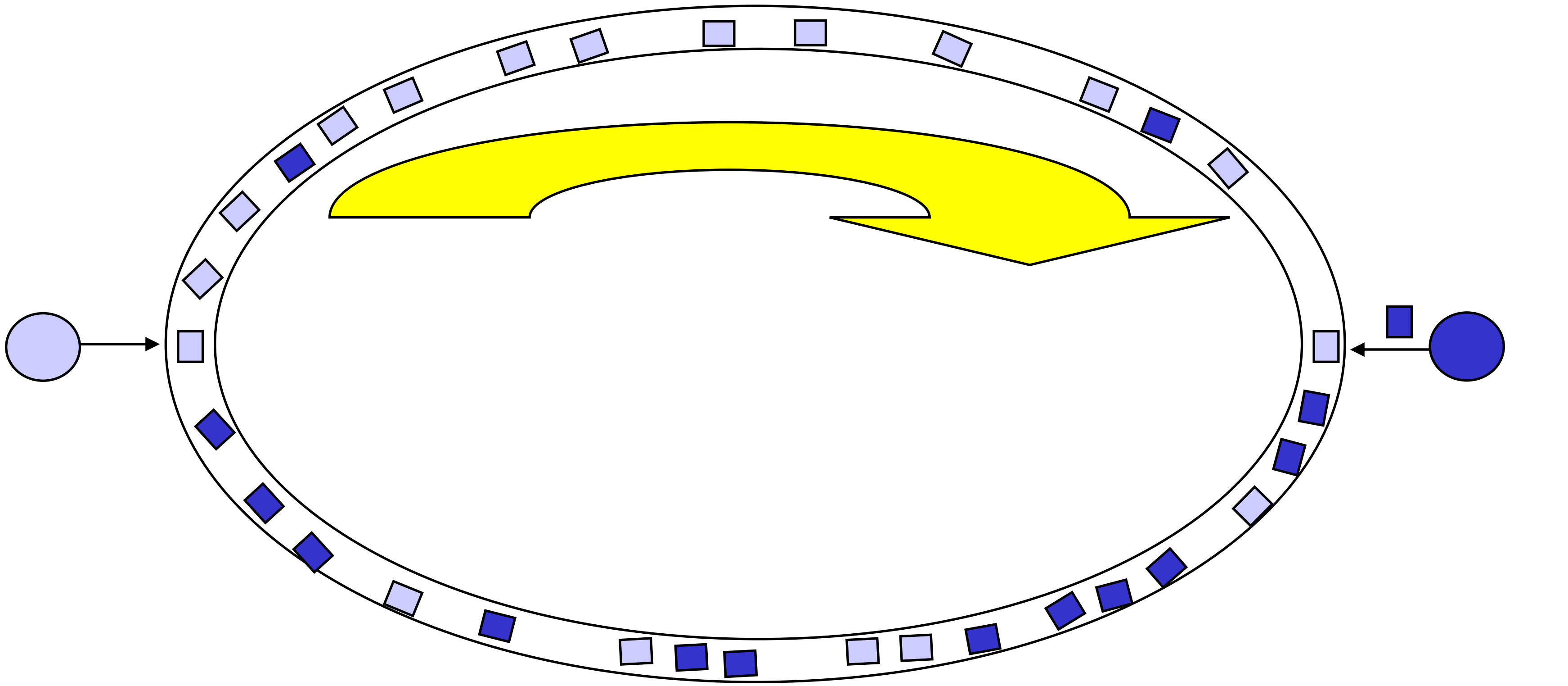
Faster Inductor

- When one inductor is faster than the other, which inductor should be first?
- Can answer mathematically (see Result 1).

Faster Inductor



Split Inductors



Split Inductors

- Now both workers will experience blocking (not just the second inductor).
- Assume that items are equally-likely to be destined for any pack station.
- As a result, 1/2 of the items will be sorted before arriving at the other station.
- $\lambda_2' = \left[\frac{1}{\lambda_2/s} - 1 + \frac{1}{1 - \lambda_1'/(2s)} \right]^{-1} s$
- $\lambda_1' = \left[\frac{1}{\lambda_1/s} - 1 + \frac{1}{1 - \lambda_2'/(2s)} \right]^{-1} s$
- $\lambda_1' = \lambda_2' = \lambda' \Rightarrow (3)$
- Result 2 tells us that SPL always does better than SBS for $\lambda < s$.

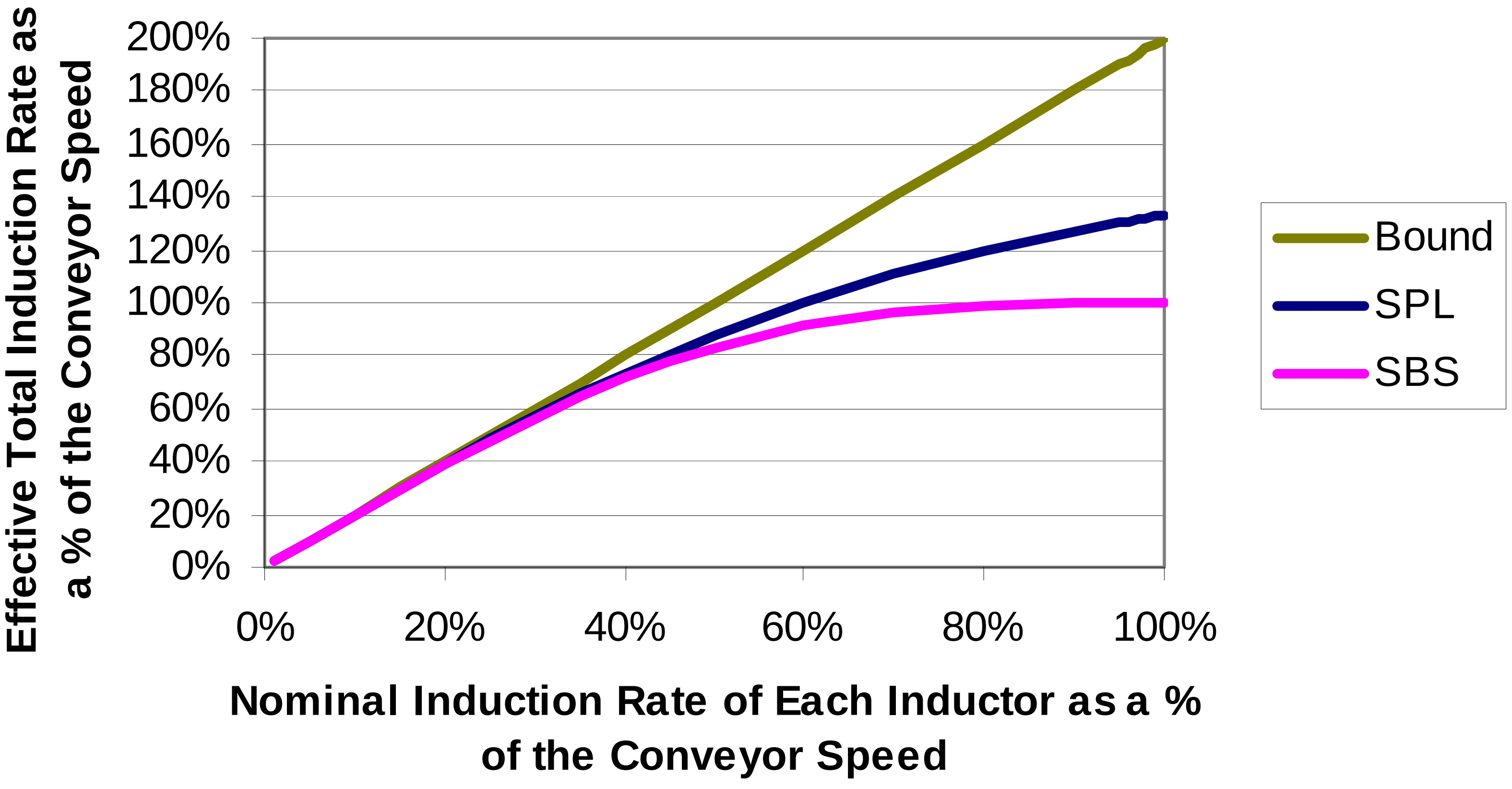
Split Results

Result 2: For two inductors each with nominal induction rate λ ($\lambda < s$), the total effective induction rate of a split system is larger than that of a side-by-side configuration ($\Lambda'_{SPL} > \Lambda'_{SBS}$).

Result 3: For two inductors working in a split configuration with nominal induction rates limited by the conveyor speed (i.e., $\lambda_i = s$, $i = 1, 2$), the total effective induction rate is expressed as $\Lambda'_{SPL} = \left(\dfrac{4}{3}\right) s$.

Result 4: For N inductors working in an equally spaced split configuration (with $B > N$ evenly distributed between the inductors) with nominal induction rates limited by the conveyor speed (i.e., $\lambda_i = s, i = 1, \ldots, N$), the total effective induction rate is expressed as $\Lambda'_{SPL} = \left(\frac{2N}{N+1}\right) s$. Moreover, $\lim_{N\to\infty} \Lambda'_{SPL} = 2s$.

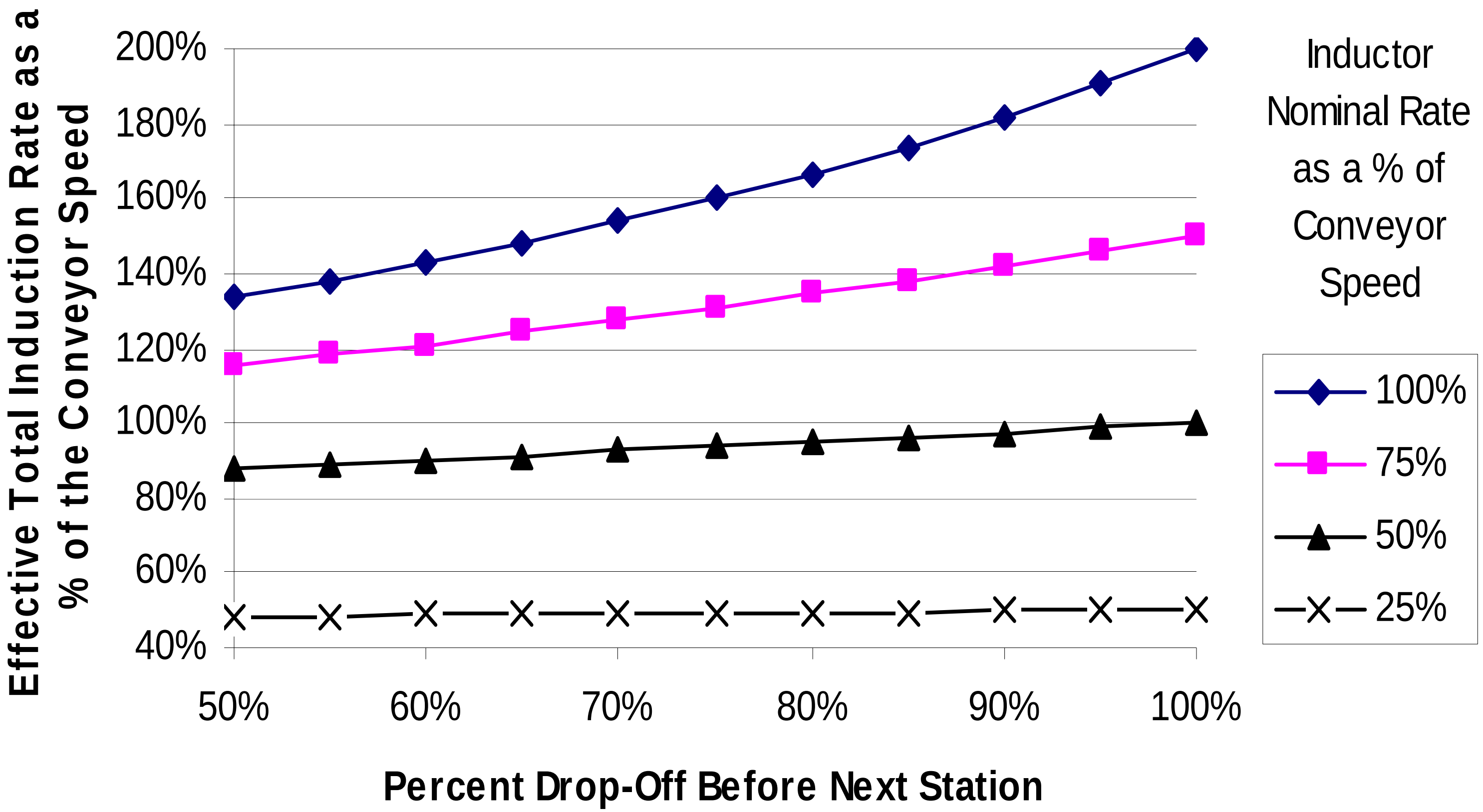
Improvement with Split



Presorting to Improve Sorter Throughput

- With two stations and no presorting:
  $$\lambda' = \left[\frac{1}{\lambda/s} - 1 + \frac{1}{1 - \lambda'/(2s)}\right]^{-1} s$$
- With two stations and presorting that leads to dropoff probability equal to d ($d > 0.5$):
  $$\lambda' = \left[\frac{1}{\lambda/s} - 1 + \frac{1}{1 - (1-d)\lambda'/s}\right]^{-1} s$$
- Note that this improves sorter throughput at the price of decreasing picking throughput.

Presorting to Improve Sorter Throughput



Approximate Model for Low Induction Variance

- Motivated by case where inductors found a rhythm.
- Did not see such "random" blocking as model would predict.
- Used an approximate queueing model based on lower bound and upper bound of throughput.
- Lower bound on throughput: Geometric Model.
- Upper bound on throughput: Finite Model.
- See paper for details (pp. 267–269).
- Approximation performed well (see Tables 1–4).

Table 1 Results for Two Side-by-Side Inductors

Simulation Experiment	AVG TBA Inductor		Total Intensity	VAR TBA Inductor		Total Induction				Simulated Half-Width 95%	% Error
	1	2		1	2	Geometric Estimate (Λ^G)	Finite Estimate (Λ^F)	Approx. Estimate (Λ^A)	Simulation Estimate		
1	5	5	40.0%	0.80	0.80	39.05	39.23	39.19	39.30	0.01	0.27%
2	5	5	40.0%	1.30	1.30	39.05	39.23	39.19	39.23	0.02	0.10%
3	5	5	40.0%	2.40	2.40	39.05	39.23	39.19	39.21	0.03	0.07%
4	5	5	40.0%	3.20	3.20	39.05	39.23	39.18	39.21	0.03	0.07%
5	5	5	40.0%	3.60	3.60	39.05	39.23	39.18	39.21	0.03	0.06%
6	4	4	50.0%	0.75	0.75	48.08	48.53	48.41	48.60	0.02	0.40%
7	4	4	50.0%	1.50	1.50	48.08	48.53	48.40	48.50	0.04	0.21%
8	4	4	50.0%	2.10	2.10	48.08	48.53	48.39	48.46	0.03	0.15%
9	4	4	50.0%	2.55	2.55	48.08	48.53	48.38	48.45	0.04	0.14%
10	4	4	50.0%	2.78	2.78	48.08	48.53	48.37	48.42	0.04	0.08%
11	3	4	58.3%	1.00	0.75	55.55	56.41	56.09	56.33	0.03	0.41%
12	3	4	58.3%	1.33	1.50	55.55	56.41	56.07	56.24	0.03	0.30%
13	3	4	58.3%	1.60	2.10	55.55	56.41	56.05	56.19	0.04	0.25%
14	3	4	58.3%	1.80	2.55	55.55	56.41	56.04	56.15	0.04	0.20%
15	3	4	58.3%	1.90	2.78	55.55	56.41	56.03	56.08	0.05	0.08%
16	3	3	66.7%	1.00	1.00	61.90	63.33	62.76	63.08	0.03	0.50%
17	3	3	66.7%	1.33	1.33	61.90	63.33	62.73	62.94	0.04	0.34%
18	3	3	66.7%	1.60	1.60	61.90	63.33	62.69	62.86	0.04	0.27%
19	3	3	66.7%	1.80	1.80	61.90	63.33	62.67	62.81	0.05	0.22%
20	3	3	66.7%	1.90	1.90	61.90	63.33	62.66	62.75	0.05	0.15%
21	2	4	75.0%	0.75	0.75	70.00	72.22	71.03	71.14	0.03	0.16%
22	2	4	75.0%	0.83	1.50	70.00	72.22	70.98	71.06	0.05	0.12%
23	2	4	75.0%	0.90	2.10	70.00	72.22	70.93	70.94	0.04	0.01%
24	2	4	75.0%	0.95	2.55	70.00	72.22	70.90	70.87	0.06	−0.05%
25	2	4	75.0%	0.98	2.78	70.00	72.22	70.88	70.86	0.06	−0.03%
26	2	3	83.3%	0.75	1.00	75.00	78.57	76.49	76.76	0.04	0.35%
27	2	3	83.3%	0.83	1.33	75.00	78.57	76.42	76.56	0.05	0.19%
28	2	3	83.3%	0.90	1.60	75.00	78.57	76.35	76.44	0.04	0.11%
29	2	3	83.3%	0.95	1.80	75.00	78.57	76.31	76.30	0.05	−0.01%
30	2	3	83.3%	0.98	1.90	75.00	78.57	76.28	76.26	0.06	−0.02%
31	2	2	100.0%	0.75	0.75	83.33	90.00	85.68	85.79	0.03	0.13%
32	2	2	100.0%	0.83	0.83	83.33	90.00	85.55	85.51	0.05	−0.05%
33	2	2	100.0%	0.90	0.90	83.33	90.00	85.45	85.31	0.05	−0.16%
34	2	2	100.0%	0.95	0.95	83.33	90.00	85.37	85.12	0.05	−0.29%
35	2	2	100.0%	0.98	0.98	83.33	90.00	85.32	85.06	0.05	−0.31%

TBA = Trays Between Attempts. Average % Error = 0.13%.
Average (Absolute) % Error = 0.18%.

Table 2 Results for Two Split Inductors

Simulation Experiment	AVG TBA Inductor 1	AVG TBA Inductor 2	Total Intensity	VAR TBA Inductor 1	VAR TBA Inductor 2	Total Induction: Geometric Estimate (Λ^G)	Total Induction: Finite Estimate (Λ^F)	Total Induction: Approx. Estimate (Λ^A)	Total Induction: Simulation Estimate	Simulated Half-Width 95%	% Error
1	5	5	40.0%	0.80	0.80	39.15	39.23	39.21	39.24	0.01	0.06%
2	5	5	40.0%	1.30	1.30	39.15	39.23	39.21	39.21	0.02	0.00%
3	5	5	40.0%	2.40	2.40	39.15	39.23	39.21	39.23	0.03	0.04%
4	5	5	40.0%	3.20	3.20	39.15	39.23	39.21	39.22	0.04	0.02%
5	5	5	40.0%	3.60	3.60	39.15	39.23	39.21	39.21	0.03	0.01%
6	4	4	50.0%	0.75	0.75	48.34	48.53	48.48	48.54	0.02	0.14%
7	4	4	50.0%	1.50	1.50	48.34	48.53	48.47	48.52	0.03	0.09%
8	4	4	50.0%	2.10	2.10	48.34	48.53	48.47	48.50	0.03	0.06%
9	4	4	50.0%	2.55	2.55	48.34	48.53	48.46	48.49	0.03	0.06%
10	4	4	50.0%	2.78	2.78	48.34	48.53	48.46	48.48	0.04	0.03%
11	3	4	58.3%	1.00	0.75	55.76	56.09	55.98	56.02	0.03	0.07%
12	3	4	58.3%	1.33	1.50	55.76	56.09	55.97	56.00	0.03	0.05%
13	3	4	58.3%	1.60	2.10	55.76	56.09	55.96	56.01	0.04	0.08%
14	3	4	58.3%	1.80	2.55	55.76	56.09	55.96	56.00	0.04	0.07%
15	3	4	58.3%	1.90	2.78	55.76	56.09	55.96	55.96	0.05	0.01%
16	3	3	66.7%	1.00	1.00	62.77	63.32	63.10	63.20	0.03	0.16%
17	3	3	66.7%	1.33	1.33	62.77	63.32	36.09	63.17	0.04	0.12%
18	3	3	66.7%	1.60	1.60	62.77	63.32	63.08	63.12	0.04	0.07%
19	3	3	66.7%	1.80	1.80	62.77	63.32	63.07	63.12	0.05	0.08%
20	3	3	66.7%	1.90	1.90	62.77	63.32	63.06	63.09	0.05	0.04%
21	2	4	75.0%	0.75	0.75	70.14	70.82	70.53	70.51	0.05	−0.02%
22	2	4	75.0%	0.83	1.50	70.14	70.82	70.51	70.49	0.04	−0.02%
23	2	4	75.0%	0.90	2.10	70.14	70.82	70.50	70.45	0.04	−0.06%
24	2	4	75.0%	0.95	2.55	70.14	70.82	70.49	70.44	0.06	−0.07%
25	2	4	75.0%	0.98	2.78	70.14	70.82	70.48	70.41	0.04	−0.10%
26	2	3	83.3%	0.75	1.00	76.22	77.35	76.77	76.80	0.04	0.04%
27	2	3	83.3%	0.83	1.33	76.22	77.35	76.75	76.74	0.05	−0.01%
28	2	3	83.3%	0.90	1.60	76.22	77.35	76.73	76.72	0.05	−0.01%
29	2	3	83.3%	0.95	1.80	76.22	77.35	76.71	76.67	0.06	−0.05%
30	2	3	83.3%	0.98	1.90	76.22	77.35	76.70	76.67	0.07	−0.04%
31	2	2	100.0%	0.75	0.75	87.69	89.90	88.47	88.49	0.04	0.02%
32	2	2	100.0%	0.83	0.83	87.69	89.90	88.42	88.41	0.06	−0.02%
33	2	2	100.0%	0.90	0.90	87.69	89.90	88.39	88.36	0.06	−0.03%
34	2	2	100.0%	0.95	0.95	87.69	89.90	88.36	88.30	0.06	−0.07%
35	2	2	100.0%	0.98	0.98	87.69	89.90	88.35	88.29	0.07	−0.07%

TBA = Trays Between Attempts. Average % Error = 0.02%.
Average (Absolute) % Error = 0.05%.

Table 3 Results for Three Side-by-Side Inductors

Simulation Experiment	AVG TBA Inductor 1	AVG TBA Inductor 2	AVG TBA Inductor 3	Total Intensity	VAR TBA Inductor 1	VAR TBA Inductor 2	VAR TBA Inductor 3	Total Induction: Geometric Estimate (Λ^G)	Total Induction: Finite Estimate (Λ^F)	Total Induction: Approx. Estimate (Λ^A)	Total Induction: Simulation Estimate	Simulated Half-Width 95%	% Error
1	5	5	5	60%	0.80	0.80	0.80	56.78	57.77	57.39	57.68	0.02	0.50%
2	5	5	5	60%	1.30	1.30	1.30	56.78	57.77	57.38	57.44	0.02	0.09%
3	5	5	5	60%	2.40	2.40	2.40	56.78	57.77	57.36	57.32	0.02	−0.08%
4	5	5	5	60%	3.20	3.20	3.20	56.78	57.77	57.35	57.27	0.03	−0.13%
5	5	5	5	60%	3.60	3.60	3.60	56.78	57.77	57.34	57.24	0.04	−0.18%
6	4	4	4	75%	0.75	0.75	0.75	68.38	70.83	69.66	70.28	0.02	0.88%
7	4	4	4	75%	1.50	1.50	1.50	68.38	70.83	69.61	69.74	0.02	0.20%
8	4	4	4	75%	2.10	2.10	2.10	68.38	70.83	69.56	69.54	0.03	−0.03%
9	4	4	4	75%	2.55	2.55	2.55	68.38	70.83	69.52	69.45	0.04	−0.11%
10	4	4	4	75%	2.78	2.78	2.78	68.38	70.83	69.50	69.35	0.04	−0.22%
11	3	3	3	100%	1.00	1.00	1.00	83.52	90.85	86.07	86.76	0.04	0.79%
12	3	3	3	100%	1.33	1.33	1.33	83.53	90.85	85.94	86.13	0.04	0.22%
13	3	3	3	100%	1.60	1.60	1.60	83.52	90.85	85.83	85.74	0.05	−0.11%
14	3	3	3	100%	1.80	1.80	1.80	83.52	90.85	85.75	85.49	0.04	−0.30%
15	3	3	3	100%	1.90	1.90	1.90	83.52	90.85	85.71	85.38	0.06	−0.39%
16	2	2	2	150%	0.75	0.75	0.75	97.62	100.00	99.96	99.10	0.05	−0.87%
17	2	2	2	150%	0.83	0.83	0.83	97.62	100.00	99.84	98.87	0.04	−0.98%
18	2	2	2	150%	0.90	0.90	0.90	97.62	100.00	99.73	98.69	0.04	−1.06%
19	2	2	2	150%	0.95	0.95	0.95	97.62	100.00	99.65	98.57	0.05	−1.10%
20	2	2	2	150%	0.98	0.98	0.98	97.62	100.00	99.60	98.57	0.05	−1.05%

TBA = Trays Between Attempts. Average % Error = −0.20%.
Average (Absolute) % Error = 0.46%.

Table 4 Results for Four Side-by-Side Inductors

Simulation Experiment	AVG TBA Inductors 1–4	Total Intensity	VAR TBA Inductors 1–4	Total Induction: Geometric Estimate (Λ^G)	Total Induction: Finite Estimate (Λ^F)	Total Induction: Approx. Estimate (Λ^A)	Total Induction: Simulation Estimate	Simulated Half-Width 95%	% Error
1	5	80%	0.80	72.62	75.70	74.05	74.82	0.02	1.03%
2	5	80%	1.30	72.62	75.70	74.02	74.22	0.03	0.27%
3	5	80%	2.40	72.62	75.70	73.96	73.80	0.03	−0.22%
4	5	80%	3.20	72.62	75.70	73.92	73.63	0.04	−0.39%
5	5	80%	3.60	72.62	75.70	73.90	73.56	0.04	−0.45%
6	4	100%	0.75	84.61	92.03	87.07	88.90	0.03	2.06%
7	4	100%	1.50	84.61	93.06	86.95	87.36	0.03	0.47%
8	4	100%	2.10	84.61	92.06	86.84	86.74	0.03	−0.11%
9	4	100%	2.55	84.61	92.06	86.76	86.42	0.04	−0.39%
10	4	100%	2.78	84.61	92.06	86.72	86.29	0.05	−0.50%
11	3	133%	1.00	95.92	100.00	98.47	98.87	0.01	0.41%
12	3	133%	1.33	95.92	100.00	98.33	98.12	0.02	−0.22%
13	3	133%	1.60	95.92	100.00	98.23	97.65	0.03	−0.59%
14	3	133%	1.80	95.92	100.00	98.14	97.39	0.02	−0.77%
15	3	133%	1.90	95.52	100.00	98.10	97.25	0.03	−0.88%

TBA = Trays Between Attempts. Average % Error = −0.02%.
Average (Absolute) % Error = 0.58%.

Conclusions

- Concepts:
  - When does it pay to split induction stations?
  - When does it make sense to presort the items?
  - When do you need to use approximate model (with queueing approximation)?
- Skills:
  - Calculate the throughput of a side-by-side system with 2 inductors (see pg. 12).
  - Calculate the throughput of a split system with 2 inductors (see pg. 12).
  - Calculate the maximum throughput of a system with N induction stations.
  - Calculate the throughput of a split system with 2 inductors and presorting (see pp. 15–16).
- Extension:
  - Alluded to more than two inductors . . . how would you modify the models?